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ABSTRACT

For several years, concerns have been growing about the mathematics achievement of California's K-12 students. In September 1995, the California Mathematics Task Force issued a report entitled "A Call to Action: Improving Mathematics Achievement for All California Students". The Task Force also provided specific recommendations for the preparation of elementary school classroom teachers and secondary school teachers of mathematics. Part I of this report focuses on the teacher preparation recommendations of the Task Force and provides current information pertaining to those recommendations. It also presents extensive data on the current standards and requirements for teacher preparation in mathematics and the teaching of mathematics. Part II provides newly compiled data on the demand, availability, and assignment of mathematics teachers. Part III provides information on the California Mathematics Initiative for Teaching. (Includes 7 appendices). (ASK)

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Recruitment and Preparation of Teachers for Mathematics Instruction: Issues of Quality and Quantity in California

B Brunsman

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Sacramento, California

October 1997

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Recruitment and Preparation of Teachers for Mathematics Instruction: Issues of Quality and Quantity in California

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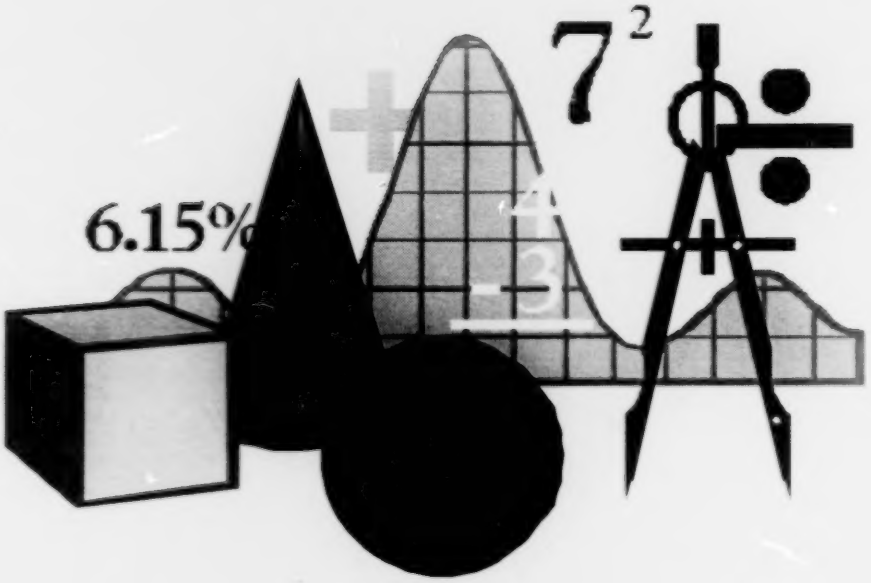
Part I
Preparation of Teachers for Mathematics Instruction: Results of Two Recent Reviews



Introduction to Part I

For several years, concerns have been growing about the mathematics achievement of California's K-12 students. California lags behind many other states in mathematics achievement scores. The United States as a whole lags behind many other nations (including nations that have fewer resources to commit to mathematics instruction). Many specialists in mathematics education have argued that the problems of student learning can be traced to inadequate standards for mathematics teachers. Still others believe that many K-12 classes in mathematics are staffed by teachers who are under-prepared in mathematics and the teaching of mathematics. The report of the Third International Mathematics and Science Study, entitled *A Splintered Vision: An Investigation of United States Science and Mathematics Education* (1997), concludes somewhat pessimistically that "it seems wishful optimism to expect our students to achieve highly in science and mathematics compared to students in other countries." California cannot afford to "settle" for low expectations, however, because evidence continues to mount that mathematical understanding will be essential for success in an increasing array of new endeavors, sectors of the economy, and high-paying jobs in the future.

In September 1995, the California Mathematics Task Force issued a report entitled *A Call to Action: Improving Mathematics Achievement for All California Students*. The Task Force consisted of university and college mathematics faculty, K-12 mathematics teachers, district administrators, parents, state and local school board members and business community members. They represented a wide range of expert viewpoints on the teaching of mathematics. The Task Force report focused on improvements in mathematics curriculum and instruction in elementary and secondary schools. The Task Force also provided recommendations for teacher preparation in mathematics. In this area, the report generally called for "mathematically powerful teachers" who can

> recognize the strengths and weaknesses of individual students (and) are able to challenge students who learn in a variety of different ways with relevant problem-solving activities, can help those who have failed to succeed during previous instruction, and can provide for the mathematics learning of students who speak languages other than English or who have physical or learning disabilities (p. 11).

For the preparation of elementary school classroom teachers, and of secondary school teachers of mathematics, the Task Force also provided specific recommendations.

Three members of the California Mathematics Task Force presented the report to the California Commission on Teacher Credentialing on December 7, 1995. The Commission discussed the Task Force findings and recommendations with the presenters. Then the Commission directed its staff to examine the Task Force recommendations that pertain to teacher preparation. The staff was specifically directed to report the extent to which the current standards and requirements for teacher certification would need to change in order to fulfill the Task Force recommendations. Finally, the Commission asked for an analysis of the fiscal and educational impact of the Task Force's recommendations pertaining to the preparation of elementary and secondary teachers of mathematics.

Part I of this report focuses on the teacher preparation recommendations of the Task Force, and provides current information pertaining to those recommendations. Part I answers the following question: How would the mathematics teacher preparation standards need to change in order to fulfill the recommendations of the Mathematics

Task Force? Because the Task Force made separate recommendations for elementary school and secondary school teacher preparation, these recommendations will be addressed separately. First, however, the report provides a general overview of current requirements and standards for teaching credentials in mathematics.

Overview of Current State Requirements and Standards for Teaching Credentials in Mathematics

Teaching Requirements and Standards

The requirements for teaching credentials are specified in the Education Code. Under state law, the Commission sets and enforces *standards* related to each requirement for each credential. The teaching credential requirements are summarized below. This report will focus on the Commission's *standards* for teaching credentials.

To earn a Single or Multiple Subject Teaching Credential, an applicant must fulfill the following requirements of the Education Code.

(1) Earn a baccalaureate or higher degree from a regionally-accredited college or university.

(2) Pass the state basic skills proficiency examination, called the California Basic Educational Skills Test (CBEST).

(3) Fulfill the state subject matter competence requirement in *one* of two alternative ways:

- Complete a Commission-approved program of subject matter study that is appropriate to the credential being sought; or
- Pass an assessment of subject matter knowledge and competence that is appropriate to the credential and accepted by the Commission.

(4) Complete a Commission-approved program of professional study that is appropriate to the intended credential and that includes courses in foundations and methods of teaching as well as supervised teaching or an internship in teaching.

(5) Complete Commission-approved courses in four topics specified by California Law:

- Methods of teaching language development and reading;
- Pedagogical uses of computers in classrooms;
- Education of exceptional students ("mainstreaming"); and
- Health education, including instruction in specified health topics.

(6) Complete one year of full-time study (or the equivalent) after the baccalaureate degree. Credential requirements (4) and/or (5) may be completed as part or all of this "fifth year of study."

For the Single and Multiple Subject Teaching Credentials, the Commission has adopted *standards* and *preconditions* for requirements (2) through (6), as outlined above. The standards and preconditions associated with requirement (3) are the primary focus of this report.

Programs of Subject Matter Preparation for Prospective Teachers

In California, subject matter preparation programs for prospective teachers are not the same as undergraduate degree programs. Postsecondary institutions govern academic programs that lead to the award of degrees, including baccalaureate degrees in mathematics. The Commission sets standards for academic programs that lead to the issuance of credentials, including the Single Subject Teaching Credential in Mathematics and the Multiple Subject Teaching Credential. At regionally accredited colleges and universities, subject matter programs for prospective teachers must fulfill the Commission's standards. Baccalaureate degree programs for undergraduate students in mathematics may or may not fulfill the Commission's standards for subject matter preparation. An applicant for a teaching credential must have earned a Bachelor's degree from an accredited institution, but the degree may be in a subject other than the one to appear on the credential. Regardless of what subject appears on the degree, completing a subject matter program that meets the Commission's standards fulfills the subject matter competence requirement for the Single or Multiple Subject Credential.

Standards and Preconditions: Definitions

The Commission defines a *standard* as a statement of program quality that must be fulfilled for initial or continued approval of a subject matter program. In each standard, the Commission has described an acceptable level of quality in a significant aspect of the subject matter preparation of mathematics teachers. The Commission determines whether a program satisfies a standard on the basis of an intensive review of all available information related to the standard by a review panel of experts. For example, the Commission uses the following standards to evaluate programs for prospective teachers of mathematics.

> *Standard 1: Program Philosophy and Purpose.* The subject matter preparation program in mathematics is based on an explicit statement of program philosophy that expresses its purpose, design and desired outcomes, and defines the institution's concept of a well-prepared teacher of mathematics. The program philosophy, design and desired outcomes are appropriate for preparing students to teach mathematics in California Schools.
>
> *Standard 16: Delivery of Instruction in the Program.* Each program utilizes multiple instructional strategies, activities and materials that are appropriate and effective for mathematics instruction. Candidates examine ways in which mathematical knowledge is transformed for use in practical applications.

The Commission defines a *precondition* as a requirement for initial and continued program approval that is based on California state laws or administrative regulations. Unlike standards, preconditions specify requirements for institutional compliance, not program quality. The Commission determines whether a program complies with the adopted preconditions on the basis of a program proposal submitted by the college or university. In the program review sequence, a program that meets all preconditions is eligible for a more intensive review to determine if the program's quality satisfies the Commission's standards.

> *Precondition 1:* Each Program of Subject Matter Preparation for the Single Subject Teaching Credential in Mathematics shall include (a) at least 30 semester units (or 45 quarter units) of core mathematics

coursework that is directly related to subjects that are commonly taught in departmentalized mathematics classes in California public schools, and (b) a minimum of 15 semester units (or 22 quarter units) of coursework that provides breadth and perspective to supplement the essential core of the program. These two requirements are elaborated in Preconditions 2 and 3.

Development of Standards for Subject Matter Preparation

For the subject matter preparation of elementary teachers and of mathematics teachers, the Commission has adopted two distinct sets of standards. The two sets of standards were developed in a similar manner, however. The Commission established panels of experts to review recent developments in mathematics education and to recommend new standards for the academic preparation of teachers. To select the panel members, the Commission's Executive Director solicited nominations of distinguished mathematics educators from colleges, universities, school districts, county offices of education, professional organizations, and other state agencies. From large numbers of nominations, the Executive Director appointed 17 professionals to the Mathematics Teacher Preparation and Assessment Advisory Panel, and 48 members to the Elementary Subject Matter Standards Advisory Panel. Both panels included university professors as well as K-12 teachers, curriculum specialists and administrators. Each panel met a number of times to create new *Standards of Program Quality and Effectiveness* that emphasize the content-based knowledge, skills and perspectives that teachers must have in order to teach mathematics effectively in public schools. Before adopting the draft standards, the Commission distributed them to mathematics educators throughout the state and requested their comments and suggestions. After an extended period for critical analysis and discussion, the Commission's professional staff collated the responses to each standard and forwarded them to the Advisory Panels. Exercising their discretion in responding to the suggestions, the Panels made several significant improvements in the draft standards. At the conclusion of their deliberations, the two Panels presented their recommended standards to the Commission for final consideration and adoption.

Qualitative Review of Subject Matter Preparation According to Standards

To request approval for a subject matter program, each institution submits a program proposal with information pertaining to each of the standards and preconditions that are associated with the particular subject area. Each program is reviewed by a panel of subject matter experts that includes professors of the subject as well as K-12 teachers, curriculum specialists and administrators in the particular subject(s) of the program. To establish a review panel, the Executive Director conducts an intensive statewide search for the most talented educators in the subject area. Panel members are selected for their expertise in the subject and their knowledge of curriculum and instruction in California public schools. Once selected, they are trained in the purpose and function of the subject matter preparation programs; the Commission's legal responsibilities in program review and approval; the standards and alternative ways they can be met; and effective review procedures, including ways to use institutional information to determine whether standards are met.

To guide program review panels in judging the quality of programs in relation to each standard, the Commission also adopts *Factors to Consider*. Within the scope of a standard, each *factor* defines a dimension along which programs vary in quality. The factors identify the dimensions of program quality that the expert Panels considered to be

important. For example, the Commission uses the following preconditions, among others, to evaluate programs on *Standard 6: Mathematics with the Use of Technology*.

- The program provides opportunities for students to use technological tools, such as computers, calculators, graphing utilities, video, and interactive programs, to learn concepts, explore new theories, conduct investigations, make conjectures, and solve problems.

- The program utilizes appropriate technological tools when providing instruction and assessing students in mathematics.

To enable a program review panel to understand a program fully, a college or university may identify additional quality factors, and may show how the program fulfills these added indicators of quality. In determining whether a program fulfills a given standard, the Commission expects the review panel to consider all of the related quality factors in conjunction with each other. In considering the several quality factors for a standard, excellence on one factor compensates for less attention to another factor by the institution. Based on these complex review procedures, the review panels make recommendations to the Commission, which approves only those subject matter programs that are recommended by the review panels.

Elementary School Teacher Preparation in Mathematics

Recommendations by the Mathematics Task Force

The California Mathematics Task Force, which was appointed by the Superintendent of Public Instruction, made one specific recommendation regarding the preparation of "mathematically powerful" elementary school teachers. They recommended that all prospective elementary school teachers be required to "complete at least two high-quality undergraduate mathematics courses encompassing the spectrum of mathematical content strands to become credentialed" (p. 12). The "strands" to which the Task Force referred are "continuous threads running throughout the curriculum, each being developed in appropriate ways in all grade levels, kindergarten through grade 12" (*California Mathematics Framework*, 1992, pp. 5-6). The *Framework* strands include (1) number, (2) measurement, (3) geometry, (4) (patterns and) functions, (5) statistics and probability, (6) logic and language, (7) algebra, and (8) discrete mathematics.

Current Standards for Mathematics Preparation of Elementary School Teachers

Three current standards for the mathematics preparation of elementary school teachers are pertinent to the Mathematics Task Force's recommendation: Standards 2, 3 and 4. Standard 3, which is entitled *Required Subjects of Study*, states:

> Each subject matter program fosters knowledge and understanding of the subjects that are commonly taught in elementary schools. Each program includes study of each subject that is required by Education Code Section 44314: language, literature, **mathematics**, science, social science, history, humanities, the arts, physical education, and human development (emphasis added).

An institution cannot meet Standard 3 unless courses in college-level mathematics are included in the program of subject matter preparation for prospective elementary school teachers. The Task Force recommendation calls for those courses to "encompass the spectrum of mathematical content strands" that are in the *Mathematics Framework*. To govern the content of undergraduate mathematics courses, the Commission has adopted the following *Factor to Consider* for Standard 3.

- The program's curriculum reflects or builds on the major themes and emphases of all adopted State Curriculum Frameworks and Model Curriculum Guides for elementary schools.

This factor emphasizes the importance of alignment with "the major themes and emphases of all adopted State Curriculum Frameworks," which includes the *California Mathematics Framework* of 1992. Accordingly, this factor specifically addresses the Task Force's recommendation that college mathematics courses for teachers encompass the strands that were introduced in the 1992 *Mathematics Framework*. In actual practice, teacher education programs that are offered by colleges and universities are reviewed by panels of subject matter experts who look for coverage of the *Framework's* mathematical strands in their review of all required math courses. Additionally, the language of this factor requires that teacher preparation programs incorporate any

changes in the curriculum strands that are introduced by the new *Mathematics Framework* that is due to be adopted in 1998.

The Task Force report also emphasized that a comprehensive K-12 mathematics program includes a balance of "basic skills, conceptual understanding, and problem solving" (see p. 2). The Commission has adopted the following additional factor for Standard 3.

- The study of mathematics develops or builds upon knowledge of fundamental mathematical skills, concepts, relationships and problem solving abilities.

It can be seen that the Commission's second factor for Standard 3 emphasizes the same "mathematical skills, concepts, relationships and problem solving abilities" that the Task Force recommended.

The Task Force also recommended that prospective elementary teachers complete *"high quality* undergraduate mathematics courses" (p. 12, emphasis added). Pertaining to the *rigor* and *quality* of coursework in mathematics, the Commission has adopted Standard 2, entitled *Character of Content,* which states:

> Each subject matter program is academically rigorous and intellectually stimulating. Throughout the program, candidates are required to read major works; analyze, evaluate, and apply ideas; express themselves skillfully; and utilize processes of inquiry.

The Task Force elaborated on their concept of "quality" in their definition of "mathematical power," which emphasized the need for teachers to possess (a) deep understanding of mathematical content and (b) conceptual understanding and problem-solving abilities related to mathematics (p. 10). The Task Force report also underscored the importance of communicating mathematical thinking effectively. Following are some of the relevant *Factors to Consider* for Standard 2.

- Candidates are required to engage in rigorous studies, and are expected to master content that challenges their intellectual capacities.

- Candidates are required to write or speak articulately and (in appropriate areas) to express themselves creatively through nonverbal means.

- Candidates have opportunities to generalize, infer, compare and contrast, make hypotheses, organize information and interpret its implications, write analytically, and use research evidence, intellectual reasoning, and personal judgment.

Overall, then, the Commission's *Standards* and *Factors to Consider* for the mathematics preparation of elementary teachers ensure that coursework in elementary subject matter programs is "high quality" by addressing the mathematical power of prospective elementary teachers, as the Mathematics Task Force recommended.

Additionally, the Commission's Standard 4, on *Depth of Study,* requires that each candidate complete a concentration or a major in a discipline or an area of study that is commonly taught in elementary or middle schools or a closely related area of study. Each concentration must consist of at least 12 semester units (or the equivalent in quarter units) of upper division courses that are coherently related to each other. Core

courses that all candidates are required to complete cannot be counted for the concentration. The concentration in mathematics, which is one of several subject areas that institutions may choose to offer, is designed to enable teacher candidates to achieve a depth of understanding in mathematics. These teachers who gain the additional mathematics expertise are then available to schools to help with curriculum planning, staff development, and the selection of materials to improve mathematics instruction.

Data on Mathematics Courses in Elementary School Teacher Preparation

By itself, the language of the Commission's *Standards* and *Factors to Consider* does not signify that the recommendations of the Mathematics Task Force are being fulfilled. The Professional Services Division of the California Commission on Teacher Credentialing compiled data about the mathematics courses that prospective elementary school teachers are currently required to complete in their undergraduate studies. The purpose of this analysis was to determine the extent to which current *Standards* and *Factors to Consider* would need to be changed in order to satisfy the Task Force's recommendations. The data for this analysis were drawn from program proposals submitted by the colleges and universities, course catalogs, and academic personnel at the institutions. Included in the analysis were mathematics courses at all 72 approved college and university elementary subject matter programs.

Appendix B contains an analysis of the mathematics courses that are required by all of the subject matter programs that have been approved according to the current *Standards* and *Factors to Consider*. Appendix C enumerates the additional mathematics courses that are required for a concentration in mathematics at the institutions that offer this option. Appendix A defines the abbreviations that appear in all subsequent appendices.

Summary of the Analytical Findings

Table 1 shows how many courses in mathematics are *required* at the 72 college and university programs that prepare undergraduate students to be elementary teachers. The number of required mathematics courses ranges from one to five courses. At many institutions, prospective teachers are required to take mathematics courses that are specified by the elementary subject matter program. At other campuses, they are allowed to select from several alternative courses in mathematics. Additionally, many institutions offer other mathematics or computer science courses as *electives* that candidates can complete in order to fulfill other program requirements.

At the largest number of institutions (43), prospective elementary teachers are *required* to complete two undergraduate courses in mathematics. The next largest group of colleges and universities (20) require candidates to complete three undergraduate mathematics courses. Additionally, six institutions require completion of four courses, and one requires elementary teaching candidates to complete five mathematics courses. Only two institutions require teachers to take one math course. In all cases, the titles and unit counts of required and elective courses are in Appendix B.

Table 1:
Numbers of Required Mathematics Courses in Subject Matter Programs for Prospective Elementary Teachers

Numbers of Required Mathematics Courses	Numbers of Institutions
1	2
2	43
3	20
4	6
5	1

Appendix C shows that fifty-one programs offer *concentrations* in mathematics. All of the programs except one require that elementary teaching candidates complete at least 12 semester units (18 quarter units) in mathematics for a concentration. These courses are specified in addition to the mathematics courses that all students must take to qualify for basic teaching credentials. Eight institutions require a *major* in mathematics to meet the concentration requirement, and two programs require a *minor* in mathematics for a mathematics concentration.

Preparation in Methods of Teaching Mathematics

The California Mathematics Task Force did not provide specific recommendations about the preparation of elementary teachers in *methods* of teaching mathematics. Nevertheless, the Commission's Professional Services Division compiled information about this important aspect of preparing teachers for instruction in Grades K-6 (see Appendix D).

Several institutions that offer professional teacher education programs for the Multiple Subject Teaching Credential were surveyed over the telephone to obtain information about required courses that deal specifically with *methods* of teaching *mathematics*. Five California State Universities (Long Beach, Northridge, Sacramento, San Bernardino and San Marcos), one University of California campus (Los Angeles), and five private institutions (Chapman University, Claremont Graduate School, Loyola Marymount University, Point Loma Nazarene College and Southern California College) were interviewed. These institutions were selected to be reasonably representative of the types and sizes of colleges and universities where credentials are earned by elementary school teachers.

The results of this analysis are presented in Appendix D (see Appendix A for the table keys). Ten of the 11 institutions require completion of at least one teaching methods course that focuses specifically on how to teach mathematics in elementary schools. At these 10 institutions the number of units of required mathematics methods ranges from one to three semester units (one to four quarter units). These courses in mathematics teaching methods are *in addition to* the required courses in the content of mathematics (described above).

Preparation of Secondary School Mathematics Teachers

Mathematics Task Force Recommendations

The Mathematics Task Force made three specific recommendations for the preparation of "mathematically powerful" secondary school teachers of mathematics.

(1) That the State Superintendent of Public Instruction require all prospective secondary school mathematics teachers to "complete the equivalent of an undergraduate minor in mathematics to become credentialed."

(2) That the above requirement "should be the primary requirement rather than the current requirement, which simply involves passing a test."

(3) "Credential programs must stress mathematics and knowledge of how to teach it" (p.12).

Current Preconditions and Standards for the Preparation of Secondary School Teachers of Mathematics

The Mathematics Teacher Preparation Advisory Panel that created the current *Mathematics Teacher Preparation Standards* and recommended them to the California Commission on Teacher Credentialing agreed with the Mathematics Task Force about the importance of mathematical power. In their Preamble to the Mathematics Program Standards, the Mathematics Teacher Preparation Panel stated:

> The Commission and the Mathematics Advisory Panel believe that the goals of mathematics education are mathematical power and literacy, which include the ability to solve problems, communicate ideas, reason, make mathematical connections, and use current technology, as well as understanding the concepts and topics that have traditionally composed the mathematics curriculum. The development of mathematical power and literacy requires a redefinition of the mathematics curriculum so mathematics is approached from a unified perspective and not as a series of disjointed topics, specific concepts, or procedures to be followed.
>
> *A successful Mathematics Subject Matter Preparation Program must be aligned with this redefinition of the curriculum. The program must approach mathematics in a unified, integrated way, and must provide opportunities for students to learn mathematics in ways that are consistent with the curriculum they will be expected to teach in their own classrooms.*

Both the *Preconditions* and the *Mathematics Teacher Preparation Standards* are related to the recommendations of the Mathematics Task Force. The *Preconditions*, which were drafted by the Mathematics Teacher Preparation Panel address recommendations (1) and (3):

> (1) Each Program of Subject Matter Preparation for the Single Subject Teaching Credential in Mathematics shall include (a) at least 30 semester units (or 45 quarter units) of core mathematics coursework that is directly related to subjects that are commonly taught in

departmentalized mathematics classes in California public schools, and (b) a minimum of 15 semester units (or 22 quarter units) of coursework that provides breadth and perspective to supplement the essential core of the program. These two requirements are elaborated in Preconditions 2 and 3.

(2) The basic core of the program shall include coursework in (or directly related to) first and second year algebra (or demonstrated proficiency), geometry, first and second year calculus, number theory, mathematical systems, statistics and probability, discrete mathematics, and the history of mathematics.

In addition to describing how a program meets each standard of program quality in this handbook, the program document by an institution shall include a listing and catalog description of all courses that constitute the basic core of the program. Institutions shall have flexibility to define the core in terms of (a) specifically required coursework or (b) elective courses related to each required mathematical subject. Institutions may also determine whether the core consists of (a) one or more distinct courses for each mathematical subject, or (b) courses that offer integrated coverage of the required subjects.

(3) Additional coursework in the program shall be designed to provide breadth and perspective to supplement the essential core of the program.

A program document shall include a listing and catalog description of all courses that are offered for the purposes of breadth and perspective. Institutions may define this program component in terms of required coursework or elective courses.

In every approved subject matter program in mathematics, all prospective secondary school mathematics teachers complete more than a minor in mathematics. Every candidate completes at least 45 semester units of mathematics instruction and instruction in the application of mathematics. Thirty units must be directly related to subjects that are taught in secondary school mathematics classes, including algebra, geometry, functions and calculus, number theory, mathematical systems, statistics and probability, discrete mathematics, and history of mathematics. These subjects are defined more clearly in Standards 7 through 14 (each standard addresses a subject). As defined in Standards 7, 8, 12, and 14, algebra, geometry, discrete mathematics, and statistics and probability match directly with the K-12 strands that have the same names. The "functions" strand is covered by the Functions and Calculus Standard (Standard 9). The "measurement" strand overlaps with the definition of statistics and probability in Standard 12. Mathematical systems (Standard 11) and number theory (Standard 10) deal with the content of the "number" and "logic and language" strands in K-12 instruction.

History of mathematics (Standard 14) is not one of the mathematics strands, but provides the study that teachers need to understand mathematics in a historical context. One of the "Goals That Support Mathematical Power" in the *1992 Mathematics Framework* (p. 23) is "Connection to History and Society" (p. 25). The Framework suggests, "Students must receive a balanced picture of the range of careers that depend on mathematics and diversity of the people who have played important roles in mathematics" (p. 25).

Assessment of Mathematical Power by Teachers of Mathematics

The *Mathematics Teacher Preparation Standards* also address recommendation (2) of the Mathematics Task Force. The recommendation says that a mathematics minor "should be the primary requirement rather than the current requirement, which simply involves passing a test." The Task Force's characterization of the current requirement is not entirely accurate for two reasons. First, the current requirement allows prospective teachers to *choose* between (1) a mathematics subject matter program, which is more extensive than a mathematics minor in coverage of the K-12 mathematics strands discussed in the *1992 Mathematics Framework*, and (2) an examination requirement. Second, the examination requirement does not "simply involve passing a test." According to the *Mathematics Teacher Preparation Standards:*

> A student who seeks to earn the Single Subject Teaching Credential in Mathematics should **demonstrate mathematical power**. To verify that mathematical power has been attained, the Commission has developed and adopted a standardized subject matter assessment in mathematics, which consists of two sections: a two-hour examination of multiple-choice questions, and a two-hour performance assessment in mathematics. In both sections, the problems require understanding of mathematical abstraction and symbolism as well as mathematical relationships (p. 37, emphasis added).

The examinations require prospective secondary school teachers of mathematics to demonstrate mathematical power. This requirement goes beyond "simply passing a test." Prospective teachers must have mathematical power in order to demonstrate it. The *Mathematics Teacher Preparation Standards* include detailed specifications for the mathematics subject matter examinations. The content areas in the test specifications are the same as those defined in the program standards: "The specifications illustrate the mathematical knowledge, skills and abilities that students should acquire and develop in a subject matter program for prospective teachers of mathematics" (p. 37). The examinations were specifically developed to match these specifications. Additionally, the Commission conducted several studies of the match between the examinations and the test specifications, and between the test specifications and the content of mathematics subject matter programs. Educators from all over California with expertise in mathematics and methods of teaching mathematics critically analyzed the standards and the test specifications.

The mathematics examinations are difficult. Cumulatively, the percent passing for the Single Subject Assessment for Teachers (SSAT) in mathematics in 1995/96 and 1996/97 was approximately 50 percent. In addition to the multiple choice examination, prospective mathematics teachers must pass a two-part constructed response problem-solving examination. The percent passing this exam was lower than that for the SSAT exam. It is unlikely that someone with only a minor in mathematics would be able to meet the examination requirement. In 1996/97, 126 people applied for Single Subject Teaching Credentials in Mathematics on the basis of the examinations, as compared to 449 people who completed approved programs of mathematics preparation in colleges and universities.

Data on Approved Programs of Mathematics Teacher Preparation

The Commission's Professional Services Division collected data about the mathematics courses that prospective secondary teachers of mathematics are required to complete as part of an approved mathematics subject matter program. The purpose of this analysis

was to determine the extent to which the current standards and preconditions would need to be changed in order to fulfill the Task Force recommendations. The data for this analysis were obtained from program documents submitted by the colleges and universities, course catalogs, and academic personnel at the colleges and universities.

Appendix E shows how many mathematics courses are required by all of the 31 single subject mathematics programs that have been approved according to the current standards (see Appendix A for the table keys). The Mathematics Task Force recommended that all secondary school mathematics teachers should be required to have the equivalent of an undergraduate minor in mathematics. The minimum number of required mathematics units[1] ranges from 38 (57 quarter units) to 64 semester units (96 quarter units). These same institutions require from 13.3 semester units (20 quarter units) to 28 semester units (42 quarter units) for a minor. In many cases mathematics subject matter programs require more mathematics units than are required for other types of mathematics majors at the same institutions (see Table 2).

Table 2:
Comparison of Mathematics Units Between Subject Matter Programs and Mathematics Majors at the Same Institutions

Number of Mathematics Units Required by the Subject Matter Program as Compared to the Mathematics Major	Numbers of Institutions
Fewer Units in the Subject Matter Program than in the Major	6
Equal Units in the Subject Matter Program and in the Major	8
More Units in the Subject Matter Program than in the Major	17

Appendix F displays the coursework required by several of the mathematics subject matter programs (Humboldt State University, California Polytechnic State University San Luis Obispo, California State University Sacramento, Biola University, National University, and California State University Los Angeles; see Appendix A for the table keys). This sample was selected to be reasonably representative of the range of units required by the mathematics subject matter programs. The programs are presented in order according to the number of mathematics units they require, with the one requiring the highest number of units first.

[1]Mathematics units are defined for the purpose of this report as units of mathematics, statistics, and computer science courses only. Some institutions require breadth courses in related subject areas (e.g. physics, engineering) that require the application of mathematics content. These types of courses were not counted as mathematics units. Consequently, some institutions are reported as requiring fewer than 45 mathematics units.

Preparation in Methods of Teaching Mathematics

Although the California Mathematics Task Force did not provide specific recommendations about the preparation of secondary mathematics teachers in *methods* of teaching mathematics, the Commission's Professional Services Division compiled information about this important aspect of preparing teachers for instruction in secondary schools. Several of the approved teacher preparation programs for the Single Subject Teaching Credential were surveyed over the telephone to learn about required *methods* courses that deal specifically with teaching *mathematics* content. The results are presented in Appendix G (see Appendix A for the table keys). Interviews were completed with five California State Universities (Long Beach, Northridge, Sacramento, San Bernardino and San Marcos), one University of California campus (Los Angeles) and five private institutions (Chapman University, Claremont Graduate School, Loyola Marymount University, Point Loma Nazarene College and Southern California College). The institutions were selected to be reasonably representative of the types and sizes of approved programs. The number of units of required coursework in mathematics teaching methods ranges from zero to six units.

Analytic Conclusions: Mathematics Teacher Preparation

In September, 1995, the California Mathematics Task Force offered important insights into the challenges that confront our schools in the field of mathematics education. The Task Force's report included several recommendations to improve the quality of mathematics instruction for K-12 students in California. It included specific recommendations pertaining to the preparation of teachers of mathematics. Among other observations, the Task Force stated, "For sustained improvement in student achievement to occur in California, every classroom mathematics teacher must know mathematics and how to teach it" (p. 11). The Task Force report went on to suggest that California's standards for teachers of mathematics are too low, and need to be raised to higher levels of expectation.

The purpose of this analysis was to determine the extent to which the current requirements for mathematics teacher preparation would need to change in order to fulfill the recommendations of the Mathematics Task Force. *The results of the analysis indicate that the current requirements in mathematics and the teaching of mathematics meet and (in some cases exceed) the recommendations of the Mathematics Task Force.* This conclusion has been substantiated with respect to elementary school and secondary school teachers of mathematics, and with respect to their subject matter preparation as well as their professional training in mathematics teaching methods.

To implement the Mathematics Task Force recommendations pertaining to the *preparation* of teachers of mathematics, no policy changes would need to occur. It does not follow, however, that all mathematics teacher certification policies should remain unchanged in the aftermath of the Mathematics Task Force report.

Millions of students in California public schools are taught mathematics by teachers who do not meet the standards that are in place. The Mathematics Task Force described many shortcomings and deficiencies in mathematics instruction, which are serious problems that must be addressed if the mathematics achievement of California students is to improve. Because the current standards for preparing teachers of mathematics fulfill the Task Force recommendations, the serious shortcomings and deficiencies in mathematics instruction therefore stem from (a) shortages of teachers who meet the standards and (b) the widespread practice of assigning other teachers to teach mathematics. In any subject of the school curriculum, the instruction of students will appear to be inadequate if the assignments of teachers are not aligned with their preparation.

Part II of this report will include available information about the recruitment, selection, and assignment of teachers to mathematics classes. Like Part I, the analysis will be based on a compilation of existing, available information about the significant problems in mathematics education that were accurately described by the California Mathematics Task Force.

Part II
Recruitment and Preparation of Qualified Teachers of Mathematics: Long-Term Problems and Potential Solutions

Introduction to Part II

Part I of this report provided extensive data about the current standards and requirements for teacher preparation in mathematics and the teaching of mathematics. The data strongly supported the conclusion that the current standards and requirements fulfill (and, in some cases, exceed) the California Mathematics Task Force's recommendations for both elementary and secondary school teachers of mathematics.

The fact that the Commission's standards and requirements for mathematics teachers meet the expectations of mathematics experts should not give rise to complacency regarding the quality of mathematics instruction that is provided to K-12 students in California. After discussing Part I of this report in October, 1996, the Commission decided to examine other possible ways in which the under-preparation of teachers could (1) be contributing to inadequate student learning of mathematics, and (2) be addressed and resolved by pursuing new policy options. The Commission therefore directed the staff to compile information about the availability of teachers who meet the Commission's standards and requirements for credentials in mathematics. Therefore, Part II provides newly-compiled data about the demand, availability, and assignment of teachers of mathematics classes.

Availability of Well-Prepared New Mathematics Teachers: Evidence of Teacher Demand and Supply in California

Demand for Secondary Mathematics Teachers

Two types of data on the demand for secondary mathematics teachers are available from the California Basic Educational Data System (CBEDS): (1) the numbers of students and teachers in secondary (7-12) classrooms, and (2) the projected numbers of teachers that would need to be hired in the next school year to teach secondary (7-12) mathematics. Table 3 contains these data for a three-year period.

The data in Table 3 confirm that the numbers of mathematics classes and teachers grew from 1992-93 to 1996-97, as did the projected numbers of new teacher hires. The growing need for secondary mathematics teachers is underscored by Table 3, but it does not portray the whole picture. If some teachers who are already assigned to teach mathematics are not fully-prepared in math or math teaching (in relation to current expectations), then the numbers of currently-assigned teachers and projected hires may *understate* the true demand for highly-prepared mathematics teachers.

Table 3:
Enrollment and Staff in Mathematics Classes (1992-93 to 1996-97)

	1992-93	1993-94	1994-95	1995-96	1996-97
Total Enrollments	1,948,868	1,981,131	2,002,918	2,062,830	2,116,921
Numbers of Classes	64,797	65,733	66,608	67,699	70,830
Average Class Sizes	29.5	29.6	29.5	29.7	29.4
Teacher FTEs	10,876	10,989	11,141	11,449	11,932
Projected New Hires	745	798	842	873	996

Notes: The data in Table 3 are from the California Basic Educational Data System (CBEDS). These data primarily reflect secondary mathematics classes because mathematics is mainly taught in self-contained classrooms in elementary schools. FTEs are full-time equivalents. Teachers submitted the percentage of their class loads that are spent teaching mathematics. Projected new hires are the numbers of teachers that would need to be hired to teach mathematics for the following school year. For example, data for the 1994-95 school year were collected in October of 1994. School principals were asked at that time to estimate how many mathematics teachers they would hire for the 1995-96 school year.

In fact, there are several reasons to suppose that the demand data in Table 3 seriously understate the need for well-prepared teachers of mathematics. The teacher qualifications that were recommended by the California Mathematics Task Force were based on the Task Force's expertise and research. Many teachers who hold California teaching credentials that authorize mathematics instruction in middle and/or high schools have not met the expectations that were articulated by the Task Force. What standards and requirements were in place when current math teachers earned their credentials?

General Teaching Credentials and Standard Teaching Credentials were issued many years before the Commission established the current standards (as endorsed by the Task Force). General Secondary Teaching Credentials authorize teachers to teach all subjects in grades 7-12, regardless of their subject matter preparation. Standard Secondary Teaching Credentials authorize teachers to teach in the subjects of their majors and/or minors. However, a college major or minor in mathematics may or may not prepare an individual to teach the mathematics curriculum in California schools. Today, Mathematics Subject Matter Preparation Programs are closely aligned with the *California Mathematics Curriculum Framework*, which was not the case for mathematics majors or minors in prior decades. Moreover, current candidates who entered Mathematics Subject Matter Programs before January, 1995, may still obtain credentials according to prior standards that were poorly aligned with the school curriculum. In 1996-97 the Commission began for the first time to receive applications for credentials that were based on current standards for subject matter preparation.

The available evidence suggests that, even if all classes in mathematics were taught by teachers with preparation and certification in mathematics, their credential standards were *not* aligned with the balanced, challenging curriculum expectations that were

recently articulated by the California Mathematics Task Force. But are all math classes taught by teachers with extensive preparation -- albeit dated preparation -- in mathematics? This question needs to be examined, not for the purpose of being critical of practicing teachers, but to ascertain the scope of the issue of under-preparation.

Supply of Secondary Mathematics Teachers

Several factors affect the supply of teachers who are qualified to teach mathematics at the middle school and high school levels. One significant factor is the volume of college students who study mathematics in depth. Table 4 shows the numbers of mathematics degrees that were conferred by California colleges and universities from 1990-91 until 1994-95. Although the numbers of mathematics degrees are rising overall, the numbers of Bachelor's degrees have declined in this recent five-year period. Compared with the holders of Bachelor's degrees in mathematics, people with Master's and doctoral degrees in mathematics are much more likely to enter professions other than teaching.

Table 4:
Mathematics Degrees Conferred by California Institutions (1990-91 to 1994-95)

Types of Institution/Degree	1990-91	1991-92	1992-93	1993-94	1994-95
Universities of California					
Bachelor's Degrees	596	581	578	518	488
Master's Degrees	159	158	172	183	129
Doctoral Degrees	118	106	120	116	110
California State Universities					
Bachelor's Degrees	519	540	529	501	494
Master's Degrees	110	103	132	100	113
Private Institutions					
(Institutions Reporting)	(29)	(33)	(28)	(31)	(32)
Bachelor's Degrees	195	184	165	193	202
Master's Degrees	96	83	114	98	135
Doctoral Degrees	28	34	33	24	65
Total Bachelor's Degrees	1,310	1,305	1,272	1,212	1,184
Total Degrees	1,821	1,789	1,843	1,735	1,736

Note: These data were provided by the California Postsecondary Education Commission (CPEC).

The prior analysis of the demand for mathematics teachers (Table 3), showed that the demand has been increasing during the last five years. Because the numbers of Bachelor's degrees have decreased over time, the numbers of potential new mathematics teachers have also declined. This decrease in the supply of people with Bachelor's degrees in mathematics means that higher and higher proportions of these individuals would have to become teachers in order to meet the rising demand for teachers trained in California. In 1992-93, the number of projected hires was 745, which was 57 percent of the 1,305 people in Table 4 who received Bachelor's degrees in mathematics from California institutions in 1991-92. In 1995-96, the number of projected hires was 873, which was 74 percent of the 1,184 individuals who received Bachelor's degrees in math in 1994-95. Moreover, these percentages do not take into account the true demand for fully-qualified mathematics teachers due to the fact that some teachers who are currently assigned to teach mathematics are under-prepared.

Table 5 displays the numbers of Single Subject Credentials in Mathematics that were awarded between 1993-94 and 1996-97. The first row of Table 5 shows the total numbers of mathematics credentials issued by fiscal year. The two following rows delineate the routes that candidates used to meet the subject matter requirement for the credential: completion of Commission-approved subject matter programs or passage of Commission-sponsored examinations.

Table 5 shows that the numbers of mathematics credentials have been declining since 1993-94. Additionally, the number of credentials awarded in mathematics in 1995-96 (673) was insufficient to meet the demand (projected hires) for the next school year (873; see Table 3). This insufficiency has increased each year since 1993-94.

Table 5:
Candidates Who Received First-Time or New Type Single Subject Credentials in Mathematics (1993-94 to 1996-97)

	Single Subject Credentials in Mathematics			
	1993-94	1994-95	1995-96	1996-97
Total Credentials Issued	748	693	673	575
Completed Program	470	475	431	449
Passed Examination(s)	278	218	242	126

Note: These data are from the Commission's workload data. The error factor is estimated to be less than 1% error.

Table 6 shows the numbers of internship credentials in secondary mathematics from 1993-94 to 1996-97. These teachers have already met the current standards for subject-matter competence in mathematics. They learn the pedagogy of mathematics teaching as part of their internship programs, and are available to work in districts that have or are participating in these programs. Even counting these interns, however, there was a shortage of 145 fully-qualified mathematics teachers (873-673-55=145) compared with the estimated number of new hires for the 1996-97 school year.

Table 6:
Candidates Who Received Single Subject Internship Credentials in Mathematics (1993-94 to 1996-97)

Types of Programs	Internship Credentials in Mathematics			
	1993-94	1994-95	1995-96	1996-97
College or University	54	38	50	43
District Interns	24	32	22	12
Total Interns	78	70	72	55

Note: These data are from the Commission's workload data. The error factor is estimated to be less than 1% error.

Summary of Tables 3-6 on Mathematics Teacher Supply and Demand. Overall, the mathematics teacher supply data in Tables 4, 5 and 6 indicate clearly that the annual supply of new teachers of mathematics is not sufficient to meet the demand for well-prepared teachers in California's K-12 classrooms, as reflected in Table 3. Before considering ways in which the problem of inadequate supply could be addressed, the report next examines further evidence that suggests the shortage of mathematics teachers may be more serious than is indicated in Tables 3-6.

Further Dimensions of the Mathematics Teacher Shortage

School districts do not have the option of not offering classes in mathematics to students who need to complete those classes. Consequently, districts cannot leave teaching positions vacant, even when there are no well-prepared applicants for those positions. Each year the shortage of well-prepared applicants (the difference between the supply and demand in Tables 3-6) is filled by putting under-prepared, under-qualified teachers into those positions. School districts may put under-qualified teachers into classrooms by using several different methods: legal out-of-credential assignments, emergency permits, waivers, and misassignments. These dimensions of the mathematics teacher shortage are explored next.

Legal Out-of-Credential Assignments

The State Education Code allows five kinds of legal out-of-credential assignments.

- Section 44256(b) allows teachers with elementary credentials to teach in departmentalized classes below grade 9 if the teachers have completed 12 semester units or 6 upper-division/graduate semester units in the subject to be taught. This option is used mostly by **middle schools**.

- Section 44258.2 allows teachers with credentials for departmentalized classes in one subject to teach another subject in departmentalized classes in grades 5-8 if the teachers have completed 12 semester units or 6 upper-division/graduate semester units in the subject to be taught. This option is used mostly by **middle schools**.

- Section 44258.3 allows teachers with elementary credentials to teach in departmentalized classes (K-8) if the teacher's subject matter competence has been verified according to a policy approved by the district's governing board. This option is used mostly by **middle schools**. (After 1/1/96, the law was changed to allow teachers with any teaching credentials to teach in grades K-12.)

- Section 44258.7 allowed teachers with any teaching credentials and special skills and preparation outside their credential authorizations to teach in the areas of their special skills, provided the assignments were approved by the local Committees on Assignments. This option was used by **middle schools and high schools**.

- Section 44263 allows teachers with any teaching credentials to teach in departmentalized classes (K-12) if the teachers have completed 18 semester units or 9 upper-division/graduate semester units in the subject to be taught. This option is used mostly by **high schools**.

For each of these five legal out-of-credential assignment options, Table 7 shows the numbers of assignments that occurred in mathematics in one third of the districts monitored by County Offices of Education each year from 1992-93 to 1994-95.

Table 7:
Legal Out-of-Credential Assignments in Mathematics by Type for One Third of School Districts Per Year (1992-93 to 1994-95)

Education Code Sections that Allow Assignments	1992-93			1993-94			1994-95		
	Elem	Mid	High	Elem	Mid	High	Elem	Mid	High
EC44256(b) (Elem. Cred. holders with 6/12 units can teach in grades K-8)	0	169	0	2	78	0	1	209	0
EC44258.2 (Sec. Cred. holders with 6/12 units can teach in grades 5-8)	0	79	0	0	51	0	0	92	0
EC44258.3* (Elem. Cred. holders can teach in grades K-8 according to policy)	0	42	0	0	37	0	0	83	0
EC44258.7 (Credentialed teachers with special skills can teach grades K-12)	1	54	48	0	47	52	0	48	54
EC44263 (Credentialed teachers with 9/18 units can teach K-12)	0	0	217	0	0	134	0	0	154
State Board Waivers	0	0	0	0	1	0	0	0	0
TOTAL MATH ASSIGNMENTS	**1**	**344**	**265**	**2**	**214**	**186**	**1**	**432**	**208**

Notes: These data were reported to the Commission by County Offices of Education. Each year represents one third of districts monitored by County Offices. State Board Waivers were two-year waivers that were granted by the Board before the Commission assumed that authority.

Abbreviations in the column titles stand for the following: Elem = elementary schools; Mid = middle schools; High = high schools.

*This Education code is used mostly by middle schools. A small number of these assignments may have been to elementary school classes.

The total assignments across the three years (1,653) is an estimate of the total number of out-of-credential assignments for 1994-95. This estimate may underestimate the actual number of these assignments, however, because out-of-credential assignments may have increased over the three year period.

Table 8 contains specific information about the types of credentials held by teachers in high school mathematics classes under Education Code 44263, which is one of the five legal "routes" to the re-assignment of teachers to mathematics classes.

Table 8:
Primary Credentials of Teachers Who Were Assigned to Teach Secondary Mathematics Under Education Code 44263

Credential Subjects	1992-93	1993-94	1994-95
Art	1	2	15
Agriculture	3	4	2
Business	24	19	9
English	7	9	7
Foreign Language	1	0	0
Home Economics	3	1	1
Industrial Technology	23	9	14
Mathematics	4	8	0
Music	18	2	0
Physical Education	39	21	33
Science	33	19	24
Social Science	29	20	22
Other Subject Areas	52	46	34
Total Teachers	217	134	154

Note: These data were reported by California County Offices of Education. Each year represents one third of districts monitored by County Offices.

Table 8 shows that the largest numbers of teachers who were assigned to teach mathematics classes pursuant to Education Code Section 44263 held teaching credentials in physical education.

Emergency Permits and Waivers to Teach Mathematics

School districts can also obtain emergency permits and waivers for under-qualified teachers to teach mathematics. Table 9 displays the numbers of emergency permits and waivers for teaching secondary mathematics classes that were granted from 1993-94 through 1996-97. For an emergency permit, the teacher must have earned a Bachelor's degree, passed the CBEST, and completed a minimum of 18 semester units or 9 upper division/graduate semester units of course-work in the subject to be taught; or the teacher must have a valid non-emergency teaching credential in another subject. In order to renew the permit, the teacher must complete six semester units toward earning a credential in mathematics. For a limited-assignment emergency permit, the teacher must have a valid teaching credential in another subject. There are no prior requirements for a waiver to teach mathematics, except that teachers must have passed or never taken the Mathematics Section of the CBEST. In fact, however, most teachers who are teaching mathematics on waivers have Bachelor's degrees and have taken at least six semester units in mathematics.

Table 9:
Emergency Permits and Waivers in Secondary Mathematics (1993-94 to 1996-97)

Types of Permits to Teach Math	1993-94	1994-95	1995-96	1996-97
New or First-Time Emergency Permits	686	475	541	669
Emergency Permit Renewals	567	627	665	720
New or First-Time Limited-Assignment Emergency Permits	156	205	160	164
Limited-Assignment Emergency Permit Renewals	71	44	51	64
Waivers to Teach Mathematics	NA[a]	29	48	NA[b]
Total Emergency Permits and Waivers	1,480[a]	1,380	1,465	1,617[b]

[a]The number of waivers in 1993-94 is not available (NA) because waivers were granted by the Department of Education at that time and no records are available that separate waivers from emergency permits.

[b]The number of waivers in 1996-97 was not available (NA) at the time this report was printed.

Table 9 indicates that the volumes of emergency/waiver teachers in mathematics have increased since 1994-95 and they are alarmingly large numbers of teachers. In a typical secondary school teaching assignment (7-12), a teacher instructs as many as 170 students every day. In all likelihood, many of the teachers in Table 9 were not assigned to teach mathematics full time (five periods per day). In fact, the numbers of mathematics classes taught by the teachers in Table 9 are not known. Overall, however, they taught mathematics to more than 54,000 students (one period of math per day) up to a possible maximum of 274,000 students (five periods of math each day) in 1996-97. It is difficult to envision a strategic policy change that would improve the effectiveness of

mathematics education in California without addressing and reducing the volumes of emergency teachers and waiver teachers of mathematics.

Extent of Teacher Misassignments in Mathematics Classes

Additionally, school districts sometimes assign under-qualified teachers to mathematics classes without authorization by law or from the Commission. Table 10 shows how many teachers were misassigned to teach mathematics in one-third of the school districts in California each year from 1992-93 to 1994-95. The total misassignments across the three years (684) is an estimate of the total number of mathematics misassignments for 1994-95. Six hundred and ninety-two of these misassignments were in secondary mathematics classrooms. These estimates may underestimate the actual number of misassignments, however, because misassignments may have increased over the three year period.

Table 10:
Numbers of Credentialed Teachers Who Were Misassigned to Teach Mathematics in One Third of School Districts Per Year (1992-93 to 1994-95)

School Levels	1992-93	1993-94	1994-95
Elementary Schools	5	3	0
Middle Schools	174	132	168
High Schools	84	60	66
Total Misassigned Teachers	263	195	234

Note: These data were reported to the Commission by County Offices of Education. Each year represents one third of districts monitored by County Offices. The data in Table 10 exclude the Los Angeles Unified School District for 1992-93 and 1993-94. Los Angeles Unified School District only reported total misassignments for those years.

If we assume that school districts would not use out-of-credential assignment methods, emergency permits, or waivers, or misassign teachers if they could hire enough well-prepared mathematics teachers, then Tables 7-10, taken together, reflect the magnitude of the mathematics teacher shortage.

Summary: Overall Dimensions of the Mathematics Teacher Shortage (7-12)

One way to estimate the demand for fully-qualified secondary (7-12) mathematics teachers for 1995-96, which is the most recent year for which all of the necessary data are available, would be to add together the projected numbers of new teachers to be hired in the 1995-96 school year (842) and the total out-of-credential assignments

(1653), emergency permits and waivers (1,380), and misassignments (684) in secondary mathematics in the 1994-95 school year. The total demand, **4,559 mathematics teachers**, assumes that fully-qualified teachers should fill all of the projected new positions as well as the positions that were filled by teachers with out-of-credential assignments, emergency permits, waivers, and misassignments in 1994-95.

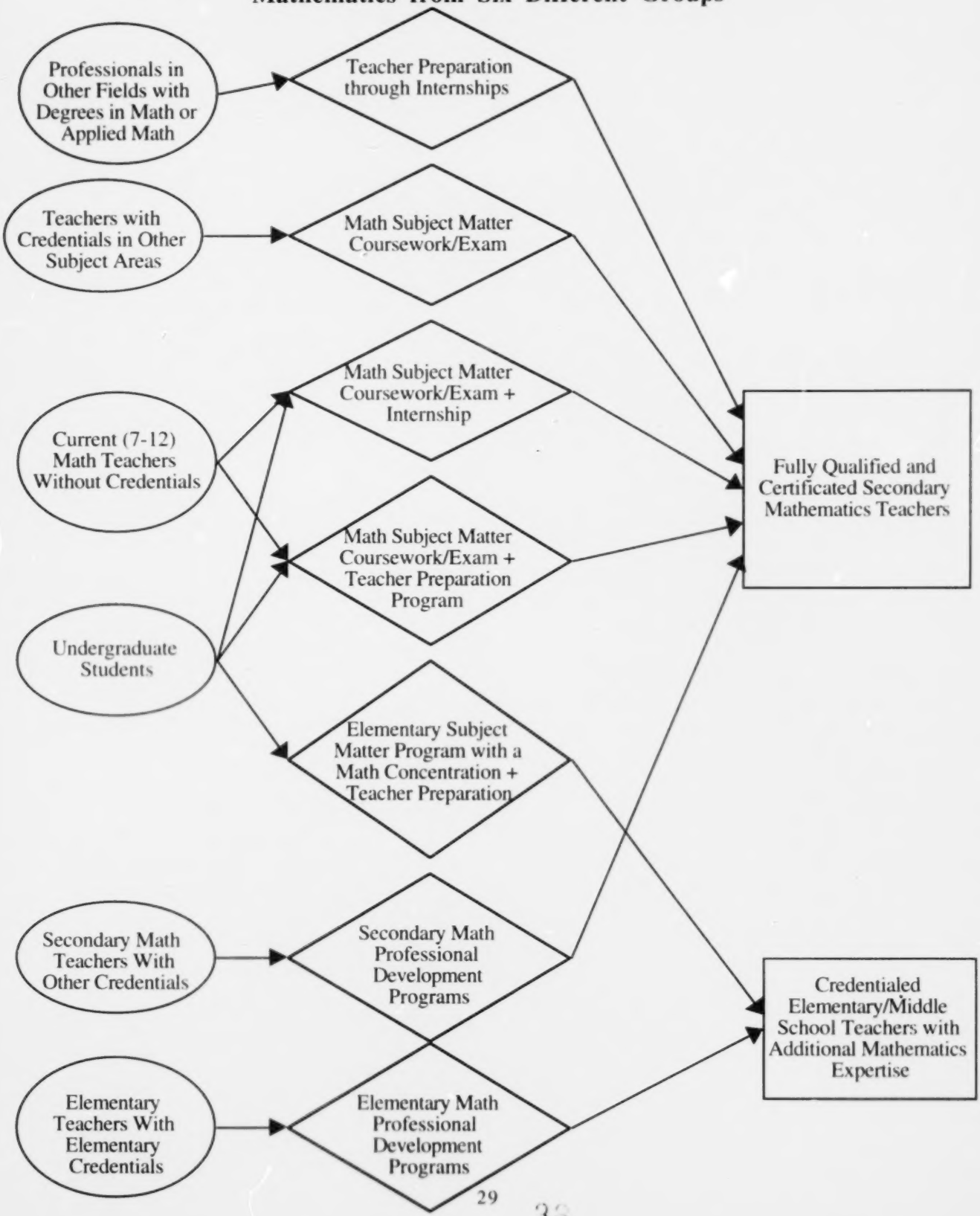
Given the fact that only 693 people earned Single Subject Credentials in Mathematics in 1994-95, and only 72 others received Internship Credentials in Mathematics in 1995-96, the *difference* between the total *demand* (4,559 teachers) and the total *supply* of new teachers (765) was **3,794 mathematics teachers** in 1995-96. In reality, the shortage of fully-qualified mathematics teachers may have been **more than 3,794 teachers** in 1995-96 because some of the 765 newly-certificated teachers may not have accepted mathematics teaching positions in K-12 schools that year.

The gap between mathematics teacher supply and demand was probably larger in 1996-97 than it was in 1995-96 because the numbers of mathematics classes in middle and high schools have been increasing (see Table 3), as have hiring projections for secondary mathematics teachers, but the numbers of Single Subject Credentials in Mathematics decreased in 1995-96 (see Table 6).

This analysis of the shortage of mathematics teachers assumes that all certificated teachers of mathematics are well-prepared to implement the challenging, balanced curriculum that was advocated by the California Mathematics Task Force and is reflected in the *California Mathematics Curriculum Framework*. For reasons that were outlined on pages 18-19, however, nearly all of the current teachers with valid credentials that authorize secondary mathematics instruction did not have to meet the standards that have recently been adopted and implemented by the Commission -- standards that were reaffirmed by the Superintendent's Mathematics Task Force. California's secondary schools, then, are challenged by a two-pronged problem of teacher shortages in mathematics: (1) very large numbers of classes being taught by instructors without sufficient preparation in mathematics, and (2) uncounted additional classes taught by instructors whose preparation occurred before the current mathematics curriculum was envisioned.

Figure 1 displays the current routes that candidates can take to become fully-prepared and credentialed teachers of secondary school mathematics. To alleviate the shortage of secondary mathematics, it will be necessary to increase the numbers of individuals who pursue these routes to full preparation in mathematics education.

Figure 1:
Alternative Routes to Full Qualification for Prospective Teachers of Mathematics from Six Different Groups


Professionals in Other Fields with Degrees in Math or Applied Math
Teacher Preparation through Internships
Teachers with Credentials in Other Subject Areas
Math Subject Matter Coursework/Exam
Math Subject Matter Coursework/Exam + Internship
Current (7-12) Math Teachers Without Credentials
Math Subject Matter Coursework/Exam + Teacher Preparation Program
Undergraduate Students
Elementary Subject Matter Program with a Math Concentration + Teacher Preparation
Fully Qualified and Certificated Secondary Mathematics Teachers
Secondary Math Teachers With Other Credentials
Secondary Math Professional Development Programs
Elementary Teachers With Elementary Credentials
Elementary Math Professional Development Programs
Credentialed Elementary/Middle School Teachers with Additional Mathematics Expertise

Preparation of Elementary Teachers in Mathematics

With increasing recognition of the great importance of early mathematics instruction, California is also experiencing a shortage of elementary school teachers who have depth of expertise in mathematics. According to the Commission's current standards for elementary teacher preparation, each prospective teacher pursues a "concentration" for depth of study in one subject field. Teachers who gain additional mathematics expertise by completing a concentration for depth of study in mathematics (see Figure 1) are now becoming available to teach in K-6 schools. In addition to being very well-prepared to provide mathematics instruction to young children, these new teachers are also well-prepared to engage in mathematics curriculum planning, staff development, and the selection of materials to improve elementary school mathematics instruction.

Recent data provided to the Commission by the coordinators of Elementary Subject Matter Programs, however, suggest that only a small number of prospective elementary teachers concentrate in mathematics at the institutions that offer mathematics as an option for depth of study. Table 11 shows the average percentages of the prospective teachers who completed elementary subject matter programs with concentrations in mathematics. Some small institutions cannot offer concentrations in mathematics because it is difficult for them to require the upper division coursework necessary to meet the subject matter standards. Middle-sized and large institutions reported that approximately five to ten percent (5-10%) of all elementary teaching candidates pursue mathematics concentrations. At one institution (California State University Chico), approximately 19% of the prospective teachers are completing math concentrations. According to a program coordinator at A CSU Chico, this impressive success is due largely to the motivational qualities of instructors in the Mathematics Department at that campus.

Table 11:
Percentages of the Undergraduates Completing Elementary Subject Matter Programs Who Concentrated in Mathematics in 1995/96

Types of Institutions	Number Offering Math Concentrations	Number Reporting	Average Percentage
California State Universities	15	8	6%
Universities of California	6	1	0%
Private Institutions	27	9	7%

Note: The data were provided by Commission-approved Elementary Subject Matter Programs.

Figure 1 also shows the current routes that candidates can take to gain expertise in elementary school mathematics. An increase in the numbers of individuals who pursue these routes to elementary mathematics expertise will be necessary to provide each elementary school with at least one teacher who is confident enough in her or his mathematical power to convey an excitement for mathematics to children and provide expertise for curriculum planning, staff development, and materials selection.

Part III
The California Mathematics Initiative for Teaching

The California Mathematics Initiative for Teaching

As Figure 1 indicates, California does not have a shortage of *routes to certification* in the field of mathematics teaching. Rather, the empirical data in this report suggest that *insufficient numbers of individuals are pursuing the several alternative routes* that are already available under current law.

To resolve the severe shortages of mathematics teachers that California schools are experiencing, the State of California could increase the numbers of fully-qualified mathematics teachers in several alternative ways. The shortage of mathematics teachers is so severe and widespread that California needs to pursue a multi-faceted strategy that addresses several interrelated problems in a coherent, coordinated effort. In such an initiative, each element needs to increase the appeal of one or more existing routes to mathematics teacher certification. Following are several initiatives that the California Commission on Teacher Credentialing considered for the purpose of making particular routes to certification as attractive as possible for mathematics teacher candidates and current teachers. The Commission, with the help of other interested groups, began a comprehensive initiative for mathematics education, called the ***California Mathematics Initiative for Teaching (CMIT)***. The overall goal of CMIT is to increase the numbers of fully qualified secondary mathematics teachers and elementary teachers with expertise in mathematics. To accomplish this goal, the Commission invited interested organizations and individuals to make recommendations and collaborate with the Commission in making CMIT successful.

Legislative Concepts Considered by the Commission

Concept One. The Commission considered the prospect of establishing new financial incentives for high school students, undergraduate students, current mathematics teachers (7-12) who do not have credentials, and teachers with credentials in other subject areas to complete the mathematics coursework and teacher education coursework that would qualify them for Single Subject Teaching Credentials in Mathematics. This option included financial assistance in the form of forgivable loans and work-study opportunities for prospective and current teachers to pursue coursework. Teachers who received financial assistance would be required to teach math for two years for every year they received assistance. Concept One also included local assistance grants to schools to form partnerships with colleges and universities to establish and improve subject matter programs or support "Future Teachers of Mathematics Clubs".

Concept Two. The Commission also considered establishing new financial incentives for high school graduates, undergraduate students, and current teachers to complete "concentrations" for depth of study in mathematics as part of subject matter preparation and professional preparation for Multiple Subject Teaching Credentials. This concept also included scholarships in the form of forgivable loans and work-study opportunities for prospective teachers to pursue mathematics coursework. Teachers who received financial assistance would be required to teach (K-6) for two years for every year they received assistance. Concept Two also included local assistance grants to schools to form partnerships with colleges and universities to establish and improve mathematics concentrations.

Concept Three. The Commission contemplated establishing new incentives for adults with backgrounds in engineering, aeronautics, business, and industry to enter internship programs for Single Subject Credentials in Mathematics. This option would have provided forgivable loans for one- to two-year internships in mathematics education. Interns would be required to teach math for two years for each year of financial assistance.

Concept Four. The Commission also discussed establishing a new staff development program for current teachers in elementary, middle, and high schools. Instruction in such a program would focus on mathematics and mathematics teaching, so participants could become qualified teachers of mathematics by meeting the current standards. This concept included fellowships for teachers to participate in intensive (six-week) summer institutes in mathematics and mathematics teaching, which would fulfill current standards for the subject matter preparation of elementary and secondary teachers. Concept Four also included local assistance grants to support experimental uses of electronic technology (interactive multi-media programs as well as interactive distance instruction) so teachers could participate without prohibitive changes in home locations.

Concept Five. In Concept 5, the Commission considered establishing a longitudinal pilot study of the effect of school-based differential pay programs for the purpose of increasing the availability of teachers of mathematics in middle schools and high schools. A maximum of twenty middle and high schools would be selected on a competitive basis with the full consent of bargaining units. Criteria of school-site success would include measured student achievement in mathematics, attendance rates, graduation rates, college matriculation rates, and the employment and retention of fully-qualified teachers of mathematics.

Comments and Suggestions from Other Organizations

The Commission circulated copies of a draft version of this report including the legislative options to a variety of interested individuals and organizations. The Commission sought comments and suggestions about the five legislative options from the Association of California School Administrators (ACSA), the Association of Independent California Colleges and Universities (AICCU), the California Federation of Teachers (CFT), the California Mathematics Council (CMC), the California Mathematics Project (CMP), the California Parent Teacher Association (PTA), the California School Boards Association (CSBA), the California Department of Education (CDE), the California State University, including the Institute for Education Reform (CSU), the California Teachers Association (CTA), and the University of California (UC), among others.

The Commission received a number of comments and suggestions about the legislative concepts. All of the organizations were supportive of at least some of the concepts. A summary of the suggestions follows.

- The CCTC should pursue legislation that will use current structures, like the Assumption Program for Loans in Education (APLE) and the California Mathematics Project (CMP) to deliver intensive staff development to current teachers (CDE, CSBA, CSU, UC).

- This effort should be coordinated with the CCTC's strategic plan to recruit new teachers, which was recently prepared in cooperation with Recruiting New Teachers, Inc." (CSU).

- The first priority for funding should be a loan assumption program for mathematics majors who are committed to teaching as a profession. Staff development for current teachers and local assistance grants to school districts to work with current subject matter programs should also be funded (UC).

- The CMP would be willing to expand current programs and create new ones to reach under-prepared mathematics teachers. The CCTC could work with them to offer staff development that would count toward supplementary authorizations in mathematics for teachers who have credentials in other subjects and would like to (or already do) teach math (CMP).

- The CCTC should employ a "carrot and stick" approach, in which monetary incentives to potential teachers are paired with reduced opportunities for school districts to assign under-qualified teachers to teach math (CSU).

- The Commission should not pursue the differential pay pilot study. Neither the CFT nor the CTA are supportive of "merit pay" for teachers based upon the subject they teach. School districts are free to negotiate different salary schedules for teachers of different subjects under current law (CFT, CTA).

- The Commission should pursue the differential pay pilot study (CSBA).

Assembly Bill 496: The California Mathematics Initiative for Teaching

Based upon comments and suggestions from numerous organizations, the Commission decided to pursue legislation to implement portions of Concepts One, Two, and Four. Assembly Bill 496 (Lempert) is sponsored by the Commission and is designed to increase the numbers of teachers who are well-prepared to teach mathematics in K-12 schools. If AB 496 is enacted, the Commission would administer the California Mathematics Initiative in Teaching (CMIT) by awarding grants to school districts and county offices of education. The grants would provide financial assistance to enable current and prospective teachers to meet mathematics teaching standards. Teachers who accept financial assistance would be required to teach mathematics in public schools for two years for each year they received assistance. The bill's incentives for current and prospective teachers would be available for elementary school teachers and secondary school teachers of mathematics.

AB 496 includes strong incentives for current teachers and prospective teachers of mathematics. The bill provides alternate routes for current teachers, who could meet California's standards by completing college or university coursework in mathematics, by passing an examination in mathematics, or by completing Commission-approved programs of intensive professional development in mathematics. Programs like the California Mathematics Project could help teachers to meet the standards. Teachers with credentials in other subject areas and emergency permit teachers would be eligible for these incentives. AB 496 also includes incentives for prospective elementary teachers to complete specialized study and field experience in mathematics.

AB 496 would appropriate $1,500,000 each year, beginning in the 1998-99 fiscal year, to the Commission for the purposes of the bill. At this level of funding, between 2,000 and 2,700 secondary mathematics teachers and 350 prospective elementary teachers could participate in CMIT each year.

For prospective secondary teachers of mathematics, AB 496 would also expand the Assumption Program of Loans for Education (APLE), which is a highly successful teacher recruitment program. AB 496 would increase the overall authorization of the Student Aid Commission for the APLE Program from 500 to 600 awards per year. The bill would designate 33 percent of all APLE awards for prospective teachers of mathematics, and would augment the program's funding by $500,000 per year beginning in 1999-2000.

The problems of student learning in mathematics can be traced to the under-preparation of teachers in mathematics and the teaching of mathematics. AB 496 would increase the numbers of teachers who are competent to teach mathematics in K-12 schools. The Commission would administer the California Mathematics Initiative in Teaching (CMIT) by awarding grants to school districts and county offices of education, who would provide financial assistance to enable current and prospective teachers to meet California's mathematics teaching standards.

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Appendix A: Key for All Tables

Department Codes

AGEC	Agricultural Economics	MAMS	Mexican American Studies
CS	Computer Science	NS	Natural Science
DS	Decision Sciences	PHY	Physics
ECON	Economics	PLANT	Plant Science
ED	Education	PSYC	Psychology
ENGR	Engineering	SSC	Social Science
HS	Health Science	STAT	Statistics

No Department Listed Mathematics

Prerequisites

P	passing score on a mathematics placement test
A	intermediate algebra
G	high school geometry
C	introductory computer course
I	introductory algebra or introductory math

Units are reported as semester units except those noted by a "Q" or "quarter" following the number, which are reported as quarter units.

Appendix B: Required and Elective Mathematics Coursework for Liberal Studies Programs Approved Under the Current Standards

Institution and Program	Courses	Upper Level	Units	Required Math Courses Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
California Polytechnic State University Pomona Prerequisites: PA	4	1	16 Q	205 Basic Concepts of Elementary Math I (4) 206 Basic Concepts of Elementary Math II (4) 207 Basic Concepts of Geometry I (4) 491 Basic Concepts of Geometry II (4)	
California Polytechnic State University San Luis Obispo Prerequisites: AG	4-5	3	15-17 Q	327 Introduction to Modern Mathematics (4) 328 Introduction to Modern Mathematics (4) 329 Math Applications to Elementary Teaching (3)	**One or Two Courses:** 118 Precalculus Algebra (4) OR 116 Precalculus Algebra (3) & 117 Precalculus Algebra (3)
CSU Bakersfield (Liberal Studies) (BCLAD) Prerequisites: PAG	2	2	10 Q	320 Introduction to Number Systems (5) 321 Basic Concepts of Geometry & Real Numbers (5)	
CSU Bakersfield (Child Development) (Child Development BCLAD) Prerequisites: PAG	3	2	15 Q	320 Intro. to Number Systems (5) 321 Basic Concepts of Geometry & Real Numbers (5) 140 Elementary Statistics (5)	
CSU Chico Prerequisites: PAG	2-4	0	6-12	50A Concepts and Structures of Math (3) 50B Concepts and Structures of Math (3)	**May choose additional math:** 112 Basic structures of math (3) 113 Intuitive Foundations of Geometry (3)
CSU Dominguez Hills Prerequisites: P	3	0	9	105 Finite Mathematics (3) 107 Foundations of the Real Number System (3) 207 Geometry and Measurement for Teachers (3)	

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Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
CSU Fresno Prerequisites: P	2	0	6-7		**One Course (general education):** Math 11 Elementary Statistics (3) 45 What is Mathematics? (3) 70 Mathematical Analysis for Life Sciences (4) 72 Elementary Mathematical Analysis II (3) 75 Mathematical Analysis I (4) AGEC 71 Agricultural Business Statistics (3) CS 20 FORTRAN Programming (4) CS 40 Introduction to Prog. and Prob. Solving (4) DS 71 Quantitative Analysis (3) ENGR 70 Engineering Computations Using C and FORTRAN (3) ENGR 71 Engineering Computations (3) HS 92 Public Health Statistics (3) PLANT 99 Applied Agricultural Statistics (3) PSYC 42 Introductory Statistics (4) **One Course:** 5 Trigonometry (3) 41 Number Systems (3) 43 Elementary Problem Solving (3) 61 Geometry and the Imagination (3) One higher level math course (3)
CSU Fullerton Prerequisites: PAG	3	2	9	303AB Fund. Concepts of elementary Math (3) (3)	One General Education math course (3)
CSU Hayward Prerequisites: P	4	3	16 Q	4021 Number Systems (4) 4022 Informal Geometry (4) 4023 Functions (4)	One General Education mathematics or statistics course(4)
CSU Long Beach Prerequisites: PAG	3-4	1-2	9-16	110 Mathematics for Elementary Teachers I (3) NS 402 Problem Solving Applications in Mathematics for Elementary Teachers (3)	**One or Two Courses:** 111 Mathematics for Elementary Teachers II (3) OR 122 Calculus I (4) & 355 Geometry (3) **Electives that meet other requirements:** 278 Computer Application in Math for Teachers (3)
CSU Los Angeles Prerequisites: PA	2	0	8 Q	100 Introduction to College Mathematics (4) 120 Elementary Mathematics for Teachers (4)	
CSU Northridge Prerequisites: P	2	1	8	210 Basic Number Concepts (4) 310 Basic Concepts of Geometry, Probability, & Statistics (4)	

Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
CSU Sacramento (Liberal Studies) (Child Development) Prerequisites: PAG	3	2	9	17 An Introduction to Exploration, Conjecture, and Proof in Mathematics (3) 107AB Fundamental Mathematical Concepts (3) (3)	**Students with a math concentration, take the following instead of 17 and 107AB:** 30 Calculus I (4) 31 Calculus II (4) 35 Introduction to Linear Algebra (3)
CSU San Bernadino (Liberal Studies) Prerequisites: P	3	1-2	12 Q	301 Basic Mathematics for Educators (4)	**One Course:** 110 College Algebra (4) 115 The Ideas of Mathematics (4) 120 Precalculus Mathematics (4) 211 Basic Concepts of Calculus (4) **One Course:** 211 Basic Concepts of Calculus (4) 212 Calculus II (4) 302 Problem Solving in Mathematics (4) **Elective that meets other requirements:** 180 Critical Thinking Through Problem Solving (4)
CSU San Marcos Prerequisites: PA	2	0	6	210 Mathematics for Elementary Teaching I (3) 212 Mathematics for Elementary Teaching II (3)	
CSU Stanislaus Prerequisites: PA	2	0	6	1030 Elementary Foundations of Mathematics I (3) 1040 Elementary Foundations of Mathematics II (3)	
Humboldt State University Prerequisites: P or A	2	0	8	107YZ Mathematics for Elementary Education (4) (4)	
San Diego State University (including Imperial Valley Campus) Prerequisites: P	4	2	12	210 Structure and Concepts of Elementary Math I (3) 211 Structure and Concepts of Elementary Math II (3) 312 Modern Elementary Mathematics (3) 313 Modern Elementary Mathematics (3)	
San Francisco State University Prerequisites: P	3	1	9	165 Concepts of the Number System for the Elementary Grades (3) 565 Concepts of Geometry, Measurement and Probability (3)	One General Education quant. reasoning course (3)
San Jose State University (Creative Arts) (Environmental Studies) (History/Social Science) (Liberal Studies) Prerequisites: PAG	2	1	6	12 Number Systems (3) 106 Intuitive Geometry (3)	
San Jose State University (Child Development)	3	1	9	12 Number Systems (3) 106 Intuitive Geometry (3) STAT 95 Elementary Statistics (3)	

Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
San Jose State University (Natural Science) Prerequisites: P	5	2	15	12 Number Systems (3) 106 Intuitive Geometry (3) SSC 15 Quant. Reasoning in the Social Sciences (3) STAT 115A Elementary Statistics (3) CS 15 Computer System Fundamentals (3)	
Sonoma State University (Hutchins School) (Environmental Studies) Prerequisites: PA	2	1	7	100 Geometry (3) 300 Elementary Number Systems, Prob. & Stat. (4)	
Sonoma State University (Mexican American Studies) Prerequisites: PA	3	2	10	100 Geometry (3) 300 Elementary Number Systems, Prob. & Stat. (4) MAMS 460 Bilingual Fundamentals of Math (3)	
Azusa Pacific University Prerequisites: P	2	1	6	53-301 Mathematical Concepts for Elementary Teachers (3)	**One Course:** 53-110 College Algebra (3) 115 Topics in College Mathematics (3)
Bethany College Prerequisites: A	4	2	12	2203 Intuitive Approach to Geometry (3) 3103 Math Concepts (3) 3333 Statistics (3) 1202 Introduction to Computers (2) ED 1201 Introduction to Computer: Education Lab (1)	
Biola University Prerequisites: none	2	0	6	111 Fundamentals of Math (3) 200 Introduction to Probability and Statistics (3)	
California Baptist College Prerequisites: I	2-3	0	8-9		One General Education mathematics course (3) **Five units of coursework from:** 133 College Algebra (3) 114 Intermediate Mathematics (3) 245 Analytic Geometry and Calculus (5)
California Lutheran University Prerequisites: P or A	2	0	7	126 Contemporary Math for College Students (4) 127 Contemporary Mathematics II (3)	
Chapman University Prerequisites: A	2	1	6	106 Mathematics Modeling (3) 303 Introduction to Statistics (3)	
College of Notre Dame Prerequisites: PAG	2	1	6	006 Geometry, Measurement, and Number Theory (3) 102 Statistics (3)	
Concordia University Prerequisites: PA	2	1	6	201 Principals of Mathematics I (3) 301 Principals of Mathematics II (3)	
Dominican College of San Rafael Prerequisites: P	3	1	10-11	40 Probability and Statistics (3) 100 Structure of the Real Number System (4)	**One Course:** 8 College Algebra (3) 10 Precalculus (3) OR 11a Calculus (4)

Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
Fresno Pacific College Prerequisites: A or G	2	2	8	110AB Principles of Mathematics (4) 130 Arith. & Algebra of the Rational Numb. Syst. (4)	
Holy Names College Prerequisites: P	2	1	6	51 Basic Concepts of Mathematics (3)	**One Course:** 123 College Geometry (3) 152 History of Mathematics (3) 155 Logic and Problem Solving (3) 160 Probability and Statistics (3) 190 Topics in Mathematics (3)
La Sierra University Prerequisites: AGC	2	0	8 Q	201 Concepts of Mathematics I (4) 202 Concepts of Mathematics II (4)	
Loyola Marymount University Prerequisites: P or I	2	2	6	306 Mathematics for Elementary Teachers I (3) 307 Mathematics for Elementary Teachers II (3)	
Master's College, The Prerequisites: P	2	0	6	200 Mathematics for Elementary Children (3) 240 Critical Thinking and Problem Solving (3)	
Mills College Prerequisites: none	1-3	0-1	NA		**One Course:** 1 Introduction to Mathematics 54 Foundations and Fundamental Concepts 1 advanced math course 1-2 additional math or computer science courses
Mount St. Mary's College Prerequisites: GI	3	0	9-10	9 Introduction to Computer Processes (3)	**One set of courses:** 50 Elementary Number Systems (3) & 51 Modern Geometry (3) 1 College Algebra and Trigonometry (3) & 3A Calculus I (4)
Pacific Christian College Prerequisites: I	3	0	9	1100 College Mathematics I (3) 1110 College Mathematics II (3) 2500 Introduction to Computers (3)	
Pacific Union College Prerequisites: I	2	0	6 Q	211 Foundations of School Mathematics (3) 212 Foundations of School Mathematics (3)	
Patten College Prerequisites: P	2	2	6	120 Math for the Liberal Arts (3) 130 College Algebra (3)	**Electives that meet other requirements:** 125 Geometry (3) 135 Statistics (3)
Pepperdine University Malibu Prerequisites: PA	2	0	7	270 Foundations of Elementary Mathematics I (4) 271 Foundations of Elementary Mathematics II (3)	
Point Loma Nazarene College Prerequisites: P or I	3	3	11	303 Problem Solving (3) 314/324 Mathematics for Elementary Teachers (4) (4) (include 1 unit each of mathematics methods)	
Saint Mary's College of California Prerequisites: GI	2	1	7	101 Fundamental Mathematical Concepts II (3.5)	1 Fundamental Mathematical Concepts I (3.5) 7 Linear Algebra (3.5) 27 Calculus (3.5)

Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
Santa Clara University Prerequisites: none	4	0	16 Q	4 Mathematics for Elementary Teachers I (4) 5 Mathematics for Elementary Teachers II (4)	**One set of courses:** 41 Nature of Mathematics (4) & 7 Calculus for Social Sciences (4) OR 6 Finite Mathematics (4) & 8 Statistics (4) OR (For math concentration): 11 Calculus and Analytical Geometry I (4) & 12 Calculus and Analytical Geometry II (4)
Simpson College Prerequisites: P or A	2	0	6	2400 Structure of the Number System (3) 2410 Mathematical Structures (3)	
Southern California College Prerequisites: PI	2	0	6	104 Mathematics for Liberal Arts Students (3)	**One Course:** 103 Basic Concepts of Arith & Intuitive Geometry (3) 116 College Algebra (3) 117 College Trigonometry (3) any upper level course
United States International University Prerequisites: P or I	2	1	8	3301 Foundations of Mathematics (4)	**One Course:** 1109 College Algebra (4) 1115 College Algebra and Trigonometry (4)
University of La Verne Prerequisites: PGI	3	1	11	489 Developmental Mathematics (4)	**One Course:** 104 College Algebra (3) 170 Mathematics in Society (3) **One Course:** 210 Introduction to Computer Utilization (4) CS 110 Intro. to Computer Sci. & Engineering (4)
University of Redlands[a] Prerequisites: P	3	0	13	CS 102 Introduction to Computers for Educators (4)	**One set of courses:** 101 Finite Mathematics (4) & 102 Principles of Algebra and Geometry (4) OR (For math concentration): 121 Calculus I (4) & 122 Calculus II (4)
University of San Diego Prerequisites: P or A	3	1	9	11 College Algebra (3) 91 Math Concepts for Elementary Teachers (3) 101 Math Concepts for Elementary Teachers (3)	

Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
University of San Francisco Prerequisites: P or A	3	0	9		**One Course:** 101 Statistical Reasoning (3) 103 Statistical Reasoning (Honors) (3) PSYC 260 Psychological Statistics (3) **Two courses:** 104 Intermediate Algebra (3) 106 Quantitative Methods for Business (3) 108 Precalculus (3) 109 Analytic Geometry and Calculus I (3)
University of Southern California[a] Prerequisites: P or A	1	0	4 Q	040 Basic Mathematical Skills (0)[b] 200 Elementary Mathematics from an Advanced Standpoint (4)	
University of the Pacific Prerequisites: A	2	1	8	161 Elementary Concepts of Math I (4)	**One Course:** 35 Elementary Statistical Inference (4) 41 Elementary Functions (4) 51 Calculus I (4)
Westmont College Prerequisites: none	2	2	6.	160 Fundamentals of Mathematics (3) 165 Fundamentals of Mathematics (3)	
Whittier College Prerequisites: P	2	0	6		**Six units:** 76 College Algebra (3) 75 Mathematics for Liberal Arts (3) 80 Statistics (4) 85 Precalculus Mathematics (3) 130 Discrete Math (3) 141AB Calculus and Analytical Geometry 4) (4) 205 Introduction to Problem Solving (4) 290AB Seminar in Mathematics Proof (1) (1)
UC Davis Prerequisites: I	2	0	6 Q	71AB Explorations in Elementary Mathematics (3) (3)	
UC Los Angeles Prerequisites: none	3	1	12 Q	38AB Fundamentals of Arithmetic (4) (4) 104 Fundamental Concepts of Geometry (4)	
UC Riverside (Liberal Studies) Prerequisites: P or I	2	1-2	8-9 Q	ED 141 Mathematics Instructional Strategies (4)	**One Course:** 4 Liberal Arts Mathematics (4) 5 Introduction to College Mathematics (5) any higher level mathematics course

Appendix B: Required and Elective Math Coursework in Elementary Teacher Preparation Programs

Institution and Program	No. of Courses	Upper Level	Units	Required Math Course Names and Numbers	Math Course Names and Numbers from Which Candidates Must Select
UC Riverside (Other Subjects) Prerequisites: none	2	0-2	8-10 Q		**Eight quarter units:** 4 Liberal Arts Mathematics (4) 5 Introduction to College Mathematics (5) STAT 40 Elements of Statistics (4) STAT 100AB Introduction to Statistics (5) (5) CS 8 Introduction to Computing (4) ED 141 Mathematics Instructional Strategies (4)
UC San Diego Prerequisites: PA	2	0	8 Q	4C Elementary Functions (4) 6A Introductory Statistics and Math Analysis (4)	
UC Santa Barbara Prerequisites: PGI	3	2-3	10-12 Q	100AB Mathematics for Elementary Teaching (3) (3)	One General Education Quantitative Relationships course (4-6) from departments that offer this course
UC Santa Cruz Prerequisites: none	2	0	10 Q	8 Mathematics for Prospective Teachers (5)	One statistics course appropriate for student's major (5)

[a]The program includes coursework in methods for teaching mathematics.
[b]This course was not counted in the total number of courses because students do not receive any credit for it.

Appendix C: Required Courses for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
California Polytechnic State University San Luis Obispo	18 quarter	**Electives:** 119 Precalculus Trigonometry (3) 124 Finite Mathematics (3) 211 Elementary Probability and Statistics (3) 212 Statistical Methods (3) 170 Theory of Equations (2) 131, 132, 133 Technical Calculus (4) (4) (4) 141, 142, 143 Calculus (4) (4) (4) 206 Linear Algebra (4) 248 Methods of Proof (4) 300 Microcomputers in Mathematics Education (3)
CSU Bakersfield (Liberal Studies)	19-21 quarter	191 Precalculus Mathematics I: College Algebra (5) 192 Precalculus Mathematics II: Elementary Functions (5) One other course (not 320 or 321) Liberal Studies 490.003 Senior seminar
CSU Chico	12	112 Basic Structure of Mathematics (3) 113 Intuitive Foundations of Geometry (3) Two other courses beyond intermediate algebra (6)
CSU Dominguez Hills	13	131 Elementary Statistics and Probability (3) 153 College Algebra and Trigonometry (4) 141 Computers for the Teaching of Mathematics (3) **One course:** CS 111 Introduction to Computers and BASIC Programming (3) 143 Finite Mathematics (3) 171 Survey of Calculus for Management and Life Sciences (4)
CSU Fresno	12	Electives
CSU Fullerton	43	Math Major

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
CSU Hayward	24-25 quarter	1304 Calculus I (4) 1305 Calculus II (4) **One course:** 2101 Elements of Linear Algebra (4) OR 2150 Discrete Structures (4) **One Course:** 2304 Calculus III (4) 2010 Statistics for Business and Economics (5) 3010 Statistical Methods in the Social Sciences (4) **2 additional upper division Mathematics electives (8)**
CSU Long Beach	13	123 Calculus II (4) **Three courses:** 310 History of Mathematics (3) 311 Topics of Enrichment in Mathematics for the Elementary Teacher (3) 317 Introduction to Abstract Mathematics (3) 340 Theory of Algebraic Equations (3) 341 Number Theory (3) ****Must choose 122 and 355 to meet liberal studies requirement**
CSU Northridge	12	104 Trigonometry and Analytic Geometry (3) 311 Basic Geometric Concepts (3) 312 Modern Algebra (3) 331 Mathematics Explorations (3)
CSU Sacramento	12	102 Number Theory (3) 108 Introduction to Formal Mathematics (3) 121 Euclidean Geometry (3) 190 History of Mathematics (3)
CSU San Bernadino (Liberal Studies)	18	Electives
CSU San Marcos	12	Electives
CSU Stanislaus	17	Electives (at least 15 upper division units)

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
Humboldt State University	13-14	108 Elementary Statistics (4) 253 Discrete Mathematics (3) 401 History of Mathematics (3) **One Course:** 105 Calculus for the Biological Sciences and Natural Resources (3) OR 109 Calculus I (4)
San Diego State University	12	121 Calculus for the Life Sciences (3) 309 LOGO in Elementary Mathematics (3) **Two courses:** 302 Basic Mathematical Concepts (3) 303 History of Mathematics (3) 357 Probability and Statistics (3)
San Francisco State University	12	**Electives:** 124 Elementary Statistics (3) 220 Calculus and Analytic Geometry I (3) 301 Exploration and Proof (3) 309 Computation in Mathematics (3) 560 Computers in the Elementary Classroom (3) 561 Mathematics Education Computer Laboratory (1) 567 Problem Solving and Discovery in Mathematics (3) 569 Mathematics Investigations: Dissection and Integration of Topics (3)
Sonoma State University (Hutchins School)	6-7	142 Discrete Structures I (3) **One course:** 107 Precalculus (4) 131 Finite Math (3) 161 Calculus I (3-4) OR 165 Elementary Statistics (4)
Sonoma State University (Mexican American Studies)	3-4	**One Course:** 141 Ethnomathematics or Studies in Modern Math 131 Introduction to Finite Mathematics (3) 165 Elementary Statistics (4) 107 Algebra and Trigonometry (4) 200 Logic and Proof (3)

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
Azusa Pacific University	19	251 Calculus I (5) 252 Calculus II (5) 315 History of Mathematics and Number Theory (3) 340 Geometry (3) 360 Probability and Statistics (3)
Biola University	12	291 Linear Algebra (3) 302 Introduction to Modern Mathematics (3) 315 Modern Algebra (3) 415 Number Theory and History of Mathematics (3) 420 Modern Geometry (3)
California Baptist College	12	403 Linear Algebra (3) 464 Fundamental Concepts of Geometry (3) 323 Modern Algebra I (3) 363 History of Math (3)
California Lutheran University	14	251 Calculus with analytic Geometry I (4) 252 Calculus with Analytic Geometry II (4) 343 Linear Algebra (3) 371 Theory of Numbers (3)
Chapman University	12	105 Precalculus II (3) 110 Calculus I (3) 111 Calculus II (3) 150 Discrete Math (3)
College of Notre Dame	12	Electives (all upper division courses)
Concordia University	12	**Electives:** 271 Calculus I (5) 272 Calculus II (5) 295 Math Notation and Proof (3) 373 Calculus III (4) 376 Discrete Mathematics (3) 387 Theory of Probability (3) 420 Number Theory (3) 425 History of Mathematics (2) 471 Linear Algebra (3) 473 Modern Algebra (3) 495 Topics in Mathematics (1)

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
Dominican College of San Rafael	14-15	11ab Calculus with Analytical Geometry (8) **Two Courses:** 11c Intermediate Calculus (4) 115 Theory of Numbers (3) 116 Introduction to Set Theory and Logic (3)
Fresno Pacific College	14	2 Precalculus (4) 3 Calculus (4) 101 Principles of Geometry (3) 103 History of Mathematics (3)
Holy Names College	12	**Option 1:** 11-13 Calculus (12) **Option 2:** 123 College Geometry (3) 152 History of Mathematics (3) 155 Logic and Problem Solving (3) 160 Probability and Statistics (3)
La Sierra University	20 quarter	**Electives:** 324 Linear Algebra (4) 345 College Geometry (4) 361/362 Numerical Methods I, II (4) (4) 415 Sets and Number Systems (4) 421/422 Abstract Algebra I, II (4) (4) 431/432 Analysis I, II (4) (4)
Loyola Marymount University	26	Math Minor (at least 12 upper division units)
Master's College, The	12-14	**General Mathematics Concentration (12 units):** 231 Calculus I (5) 232 Calculus II (5) 361 Probability and Statistics II (4) CS 111 Computer Programming I (4) CS 112 Computer Programming II (4) PHY 251 General Physics (4) **Mathematics Concentration (12 units):** 231 Calculus I (5) 232 Calculus II (5) 233 Calculus III (3) 378 Number Theory and the History of Mathematics (3)

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
Mount St. Mary's College, Chalon	14-15	3AB Calculus I (4) (4) 4A Calculus II (3) **One Course:** 4B Calculus II (3) any upper division class
Pepperdine University Malibu	12	104 College Algebra and Trigonometry (4) 214 Calculus for Business and Economics (4) CS 250 Computing for Business (4)
Point Loma Nazarene College	12	145 Single Variable Calculus (4) 203 Introduction to Statistics (3) 223 Discrete Mathematics (3) 402 Topics in Geometry (2)
Santa Clara University	22-23 quarter	**Two or three courses:** 6 Finite Math for Social Sciences (4) 8 Introduction to Statistics (4) 13 Calculus and Analytical Geometry III (4) 21 Calculus and Analytical Geometry IV (4) 32 Mathematical Logic (4) 51 Discrete Mathematics (4) 52 Introduction to Abstract Algebra (4) 53 Linear Algebra (4) **Two or three courses:** 101 A Survey of Geometry (5) 122 Introduction to Probability (5) 170 Development of Mathematics (5) 172 Problem Solving (5) 175 Theory of Numbers (5) 176 Combinatorics (5) 177 Graph Theory (5)
Saint Mary's College of California	15.5-17.5	Math Minor

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
Simpson College	12	2430 Calculus I (3) 3630 Math History (3) **Two Courses:** 2530 Calculus II (3) 4630 Geometry (3) 3230 Modern Algebra (3) 3530 Math Statistics with Probability (3) 4230 Number Theory (3)
Southern California College	14	180-181 Calculus I, II (8) required **Two Courses:** 280 Calculus III (3) 330 Number Theory (3) 370 History of Mathematics (3) 420 Higher Geometry (3)
University of LaVerne	12	201 Calculus I (4) **Two Courses:** 327 Discrete Mathematics (4) 351 Probability (4) 361 FORTRAN (4) 482 History of Mathematics (4)
University of Redlands	16	Any 16 upper division units in mathematics
University of San Francisco	36 to 49	Math Major
University of Southern California	16	125 Calculus I (4) 126 Calculus II (4) **Two Courses:** 430 Theory of Numbers (4) 434 Geometry and Transformations (4) 450 History of Mathematics (4)

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
University of the Pacific	12	**Non-Concurrent with Math Minor:** 41 Elementary Functions (4) 51 Calculus I (4) **One Course:** 53 Calculus II (4) 73 Operations Research Models (4) 130 Topics in Applied Statistics (4) 162 Elementary Concepts of Math II (4) CS 47 Fundamentals of Math and Computer Science (3) OR CS 51 Introduction to Computer Science (3) **Concurrent with Math Minor:** 53 Calculus II (4) 49 Introduction to Abstract Mathematics (4) **One Course:** 37 Probability and Statistics (4) 39 Probability with Applications to Statistics (4)
Westmont College	12	10 Calculus (4) 20 Linear Algebra (4) 145 Number Theory (2) 155 History of Mathematics (2)
Whittier College	49	Math Major
UC Davis	73 quarter	Math Major
UC Los Angeles	73 quarter	Math Major

Appendix C: Courses Required for Math Concentrations by Prospective Elementary Teachers

Institution and Program	Concentration Units	Math Concentration Courses
UC Riverside (Liberal Studies)	24 quarter	131 Linear Algebra and Matrix Theory (4) 132 Linear and Modern Algebra (4) **Pure Mathematics Option** **Electives (16 units):** 133 Geometry 136 Introduction to the Theory of Numbers 138AB Introduction to Differential Geometry 144 Introduction to Set Theory 145AB Introduction to Topology 151ABC Advanced Calculus 171 Introduction to Modern Algebra 172 Modern Algebra **Applied Mathematics Option** **Electives (16 units):** 146ABC Ordinal and Partial Differential Equations 165ABC Introduction to Complex Variables 149ABC Probability and Mathematical Statistics **Statistics Option** **Electives (24 units):** STAT 130 Sampling Surveys 140 Nonparametric Techniques 147 Introduction to Statistical Computing 150 Basic Computer Methodology: FORTRAN 157 Statistical Computer Packages 160ABC Elements of Probability and Stat. Theory 161 Introduction to Probability Models 170ABC Linear Models
UC San Diego	68 quarter	Math Major
UC Santa Barbara	40 to 52 quarter	Math or Statistics Major
UC Santa Cruz	65 quarter	Math Major

Appendix D: Required Mathematics Methods Coursework for Some Multiple Subject Teacher Preparation Programs

Institution Name	Units Specific to Math	Mathematics Methods Course Names and Numbers
CSU Long Beach	3	ED 462 Teaching Mathematics in Culturally and Linguistically Diverse Classrooms (3)
CSU Northridge	1.5	ED 580M Elementary School Curriculum (3)[a]
CSU Sacramento	2	ED 304 Curriculum and Instruction in elementary School Mathematics (2)
CSU San Bernadino	4 quarter	ED 441 Mathematics Curriculum and Methods in the Elementary School (4)
CSU San Marcos	2-3	ED 543 Mathematics Education in Elementary Schools (3) **OR** ED 542 Content Area Methods II (4)[a]
Chapman University	1.5	ED 507 Teaching and Learning in the Elementary Classroom I (3)[a] **OR** ED 540 Teaching and Learning in the Culturally Diverse Classroom (3)[a]
Claremont Graduate School	1.5	ED 345 Elementary Math/Science Curriculum (3)[a]
Loyola Marymount	1-2	ED 505 Mathematics Methods for Elementary Teachers (2) OR ED 634 Elementary School Curriculum and Methods (3)[b]
Point Loma Nazarene College	2	314/324 Mathematics for Elementary Teachers (4) (4) (include 1 unit each of mathematics methods)
Southern California College	0	
UC Los Angeles	1 quarter	ED 318AB Principles and Methods for Multiple Subject Instruction (2) (2)[c]

[a] Half of the units associated with this course can be attributed to methods specific to mathematics.

[b] One third of the units associated with this course can be attributed to methods specific to mathematics.

[c] One fourth of the units associated with these courses can be attributed to methods specific to mathematics.

Appendix E: Subject Matter Programs for the Single Subject Credential in Mathematics

Institution Name	Number of Courses	Total Units	Upper Level Units	Essential Units	Breadth Units	Min. Units for Minor[a]	Min. Units for Major[a]
Cal Poly Pomona[b]	21[a]	78 quarter[a]	40 quarter[a]	?	?	41 quarter	78 quarter
Cal Poly San Luis Obispo[b]	20	76 quarter	44 quarter	54 quarter	22 quarter	30 quarter	75 quarter
CSU Bakersfield[b]	15	75 quarter[a]	55 quarter[a]	?	?	20 quarter	65 quarter
CSU Chico	13	48	21	33	15	21	48
CSU Fresno	13	53	33	37	16	20	43
CSU Fullerton	15[a]	49[a]	27[a]	34[a]	15[a]	25	43
CSU Hayward[b]	17-19	65-73 quarter	32 quarter	51 quarter	14-22 quarter	28 quarter	68 quarter
CSU Long Beach[b]	17	53	35	33	20	24	48
CSU Los Angeles	16	59 quarter	37 quarter	56 quarter	3 quarter	32 quarter	78 quarter
CSU Northridge	14-15	48	23	39	9	28	53
CSU Sacramento	16	48	27	36	15	20	47
CSU San Bernardino[b]	16-18[a]	62-70 quarter[a]	44-46 quarter[a]	?	?	36 quarter	74 quarter
CSU San Marcos	15	49	33-36	34	15	25	50
CSU Stanislaus	15	56	31	33	23	20	51
Humboldt State University	16-17	55	27	35	20	27	43
San Diego State University	14	46	24	31	15	20	46
San Francisco State Univ.	17	49	31-37	33	16	24	42
San Jose State University [bd]	12-15	39-52	27-39	33-37	6-15	18	46
Sonoma State University	15	50	19	36	14	20	45
Azusa Pacific University [b]	11	38	28	38	0	25	47
Biola University	14	48	24	33	15	27	48
Concordia University	15	47	31	31	16	21-23	36
Loyola Marymount University	20	59	30	42	17	26	54
National University	14	60 quarter	60 quarter	45 quarter	25 quarter	30 quarter	60 quarter
Pacific Union College	20	67 quarter	28 quarter	60 quarter	7 quarter	30 quarter	45 quarter
St. Mary's College	17	54.5	30.5	31	23.5	20.5	47.5
University of San Diego	15	47	26	32	15	18	40
University of San Francisco[c]	13	39	24	30	9	18	39
University of the Pacific[b]	14	51	21	35	16	23	36

Appendix E: Subject Matter Programs for the Single Subject Credential in Mathematics

Institution Name	Number of Courses	Total Units	Upper Level Units	Essential Units	Breadth Units	Min. Units for Minor[a]	Min. Units for Major[a]
Westmont College[b]	15	50	25	40	10	24	42
UC San Diego[c]	24	96 quarter	76 quarter	48 quarter	22 quarter	28 quarter	76 quarter

Note. Only mathematics, statistics, and computer science courses were counted in the number of courses and course units listed in the table. Many of the programs require courses in related areas of study (e.g., physics, logic, engineering, economics) in addition to the mathematics courses.

[a] Information from course catalog

[b] The Single Subject Mathematics Program is offered as a mathematics major.

[c] A mathematics major is required for the program.

[d] The program includes coursework in methods for teaching mathematics.

Appendix F: Required and Elective Mathematics Coursework in Some Subject Matter Programs for the Single Subject Credential in Mathematics

Institution Name	Course Names and Numbers: Core	Course Names and Numbers: Breadth and Perspective
Humboldt State University (55 units)	109 Calculus I (4) 110 Calculus II (4) 210 Calculus III (4) 241 Elements of Linear Algebra (2) 253 Discrete Mathematics (3) 343 Abstract Algebra (4) 371 Geometry (3) 340 Number Theory (3) 322 Intermediate Statistics (4) 301 Mathematics and Culture (3)	344 Linear Algebra (3) 407 Senior Seminar **One Course:** 361 Introduction to Mathematical Models (4) 415 Introduction to Real Analysis (3) 474 Graph Theory (3) **One Course:** CS. 131 Introduction to Computer Science (4) CS 155 FORTRAN Programming (3) **Strongly Recommended:** 481 Workshop in Tutoring Mathematics (1)
Cal Poly San Luis Obispo (76 quarter units)	141 Calculus I (4) 142 Calculus II (4) 143 Calculus III (4) 241 Calculus IV (4) 206 Linear Algebra I (4) 242 Differential Equations (4) 341 Number Theory (4) 419 History of Mathematics (4) 442 Euclidean Geometry (4) 443 Modern Geometry (4) 481 Modern Algebra I (4) STAT 321 Statistical Analysis (3) STAT 322 Statistical Analysis (4) **One course:** 335 Graph Theory (3) 336 Combinatorial Methods (3)	248 Methods of Proof in Mathematics (4) 300 Microcomputers in Mathematics Education (3) 424 Organizing and Teaching Mathematics (4) 459 Undergraduate Seminar (2) 461/462 Senior Project (3) (2) CS 118 Fundamentals of Computer Science (4)

Appendix F: Required and Elective Math Coursework in Subject Matter Programs (Single Subject)

Institution Name	Course Names and Numbers: Core	Course Names and Numbers: Breadth and Perspective
CSU Sacramento (48 units)	30 Calculus I (4) 31 Calculus II (4) 32 Calculus III (4) 102 Number Theory (3) 108 Introduction to Formal Mathematics (3) 110A Modern Algebra (3) 121 College Geometry (3) 130A Functions of a Real Variable (3) 190 History of Mathematics (3) 193 Capstone Course (3) STAT 1 Introduction to Statistics (3)	35 Introduction to Linear Algebra (3) 45 Differential Equations (3) 110B Modern Algebra (3) 130B Functions of a Real Variable (3) CS 1 Introduction to Computer Science (3)
Biola University (48 units)	105 Analytic Geometry and Calculus I (4) 106 Analytic Geometry and Calculus II (4) 112 Discrete Structures (3) 131 Classical Algebra and Geometry (3) 205 intermediate Calculus (4) 315 Modern Algebra (3) 331 Probability (3) 332 Statistics (3) 415 Number Theory and the History of Mathematics (3) 420 Modern Geometry (3)	**Fifteen units:** 291 Linear Algebra (3) 305 Advanced Calculus (3) 321 Numerical Analysis (3) 333 Operations Research (3) 410 Topics in Advanced Calculus (3) 435 Mathematics for the Physical Science (3) 440 Complex Variables (3) 450A Topics in Abstract Algebra (3) 480 Research Seminar (1-3) CS 105 Introduction to Computer Science (3)
National University (60 quarter units)	325 Discrete Mathematics (5) 411 Number Theory (5) 412 History of Mathematics (5) 416 Algebraic Structure (5) 417 Foundations of Geometry (5) 418 Statistical Analysis (5) 432 Advanced Calculus (5) 435 Linear Algebra (5) 438 Applied Mathematical Modeling (5)	**25 quarter units including:** 410 Computer Technology in the Math Classroom (5) 450 Mathematical Portfolio Project Course (5) **At least one of the following:** 436 Differential Equations (5) 440 Numerical Analysis (5) 450BC Mathematical Project Courses (5) (5) **Additional Electives**

Appendix F: Required and Elective Math Coursework in Subject Matter Programs (Single Subject)

Institution Name	Course Names and Numbers: Core	Course Names and Numbers: Breadth and Perspective
CSU Los Angeles (59 quarter units)	206-209 Calculus I-IV (4) (4) (4) (4) 255 Matrix Theory (4) 325 Math Notation and Proof (4) 384 Discrete Mathematics (4) 430 Modern Geometry (4) 455 Modern Algebra (4) 465 Advanced Calculus I (4) 474 Probability (4) 446 Number Theory (4) 320 History of Mathematics (4) 490 Senior Seminar (4)	398 Observation and Tutoring (1) **One Course:** CS 201 Introduction to Programming (4) CS 290 Introduction to FORTRAN Programming (2) **12 units in related fields which may include:** 215 Differential Equations (4) 420 Mathematical Logic (4) 456 Modern Algebra II (4) 466 Advanced Calculus II (4) 470 Numerical Analysis I (4) 472 Linear Programming (4) 475 Introduction to Mathematical Statistics (4) ECON 209 Applied Business & Economics Statistics I (4) ECON 309 Applied Business & Economics Statistics II (4) ECON 409 Applied Business & Economics Statistics III (4) CS 240 Computer Organization and Assembly Language (4) CS 292 C Programming (4) CS 340 Assembly Language and Systems Programming (4) CS 201 Introduction to Programming (4) CS 290 Introduction to FORTRAN Programming (2)

Note. Only mathematics, statistics, and computer science courses are listed. Many of the programs require courses in related areas of study (e.g., physics, logic, engineering, economics) in addition to the mathematics courses listed.

Appendix G: Required Mathematics Methods Coursework for Some Single Subject Teacher Preparation Programs

Institution Name	Units Specific to Math	Course Names and Numbers
CSU Long Beach	3	ED 450M Curriculum and Methods in Teaching Mathematics (3)
CSU Northridge	3	ED 525MA Methods of Teaching Mathematics (2) ED 525MAL Methods of Teaching Mathematics in Secondary School Lab (1)
CSU Sacramento	3	ED 386 Secondary School Mathematics (3)
CSU San Bernadino	4 quarter	ED 441 Secondary School Curriculum and Instruction II (4) **OR** 499 Teaching Mathematics (4)
CSU San Marcos	2	ED 541 Disciplinary and Interdisciplinary Applications (3)
Chapman	0	
Claremont Graduate School	3	ED 341 Secondary Mathematics Curriculum (2)[a] ED 342 Interdisciplinary Secondary Math/Science I (2)
Loyola Marymount	3	ED 588 Mathematics Methods for Secondary Teachers (3) **OR** ED 630 Mathematics Methods for Secondary Teachers (3)
Point Loma Nazarene College	4.5-6	ED 412 Methods of Teaching in Secondary Schools (3)[b] ED 414 Principles and Curriculum in Secondary Schools (3)
Southern California College	0	
UC Los Angeles	3 quarter	370 Teaching Mathematics (3)

[a] Half of the units associated with this course can be attributed to methods specific to math.

[b] ED 412 varies from 1.5 to 3 units specific to math depending upon student needs and faculty availability.